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VOLUME XXII NEW SERIES, NO. 158

JOURNAL
OF THE
AMERICAN STATISTICAL ASSOCIATION

JUNE, 1927

CONTENTS

WEALTH DISTRIBUTION IN THE CONTINENTAL UNITED STATES AT THE CLOSE OF 1921. By WILLFORD I. KING 135

THE SIZE OF AMERICAN FAMILIES IN THE EIGHTEENTH CENTURY. By ALFRED J. LOTKA 154

PRICE INDEX OF THE NATIONAL BANK OF COMMERCE IN NEW YORK. By E. M. MILLER and F. J. CAVANAUGH 171

A MEASURE OF STABILITY WITH A DISCUSSION OF CERTAIN OF ITS IMPLICATIONS. By HARRY S. WILL 184

FACTORS AFFECTING CHANGES IN SHORT TERM INTEREST RATES. By W. RANDOLPH BURGESS 195

ANALYSIS OF A U-SHAPED FREQUENCY DISTRIBUTION. By PAUL R. RIDER 202

NOTES:

PROBABLE INFERENCE, THE LAW OF SUCCESSION, AND STATISTICAL INFERENCE. By EDWIN B. WILSON 209

MEASUREMENT OF VARIATIONS IN SEASONAL DISTRIBUTION. By J. C. CLENDENIN 213

REPORT TO DATE OF THE WEEKLY INDEX NUMBER. By IRVING FISHER 221

CLASSIFICATION OF MEMBERS OF AMERICAN STATISTICAL ASSOCIATION ON BASIS OF DUTIES AND INTERESTS. By WILLFORD I. KING 224

ANSWER TO THE REVIEW OF "AMERICAN VILLAGERS" (227); COMMENT ON THE REVIEW "BUSINESS ANNALS" (228); A RETRACTION (229); AN ADDITION TO THE NEW COMPOSITE INDEX OF WAGES IN THE UNITED STATES (229); PROGRESS OF THE CENSUS (230); CAN DEPENDENCY AND RELIEF BE MEASURED? (231); FORECASTING THE VOLUME OF ORDERS OR SALES (235); MISCELLANEOUS NOTES (238); MEMBERS ADDED (242).

REVIEWS 247

RECENT LITERATURE 265

PUBLISHED QUARTERLY BY THE
AMERICAN STATISTICAL ASSOCIATION
PUBLICATION OFFICE: RUMFORD PRESS, CONCORD, N. H.
EDITORIAL OFFICE: COLUMBIA UNIVERSITY, NEW YORK CITY

Price $1.50 per copy **$5.00 per annum**

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REVIEWS: Hardy: *Tax-Exempt Securities and the Surtax*, by R. S. Tucker (247); Schacht: *Die Stabilisierung Der Mark*, by E. A. Goldenweiser (249); McGuire: *Italy's International Economic Position*, by J. W. Angell (252); Gillman: *Rent Levels and Their Causes*, by Margaret L. Stecker (255); Moulton and Pasvolsky: *World War Debt Settlements*, by R. B. Warren (257); Sloan: *Security Speculation — The Dazzling Adventure*, by D. S. Roswell (258); Fisher: *Wages and Their Regulation in Great Britain since 1918*, by A. H. Hansen (261); Lindsay: *Social Progress and Educational Waste*, by D. S. Thomas (262); Menzler: *The Future of the Actuarial Profession*, by E. W. Kopf (263).

RECENT LITERATURE 265

Published quarterly by the American Statistical Association

RUMFORD PRESS BUILDING, CONCORD, N. H.

Editorial Office, New York City

Entered at the post-office, Concord, N. H., as second-class mail matter

Acceptance for mailing at the special rate of postage provided for in Section No. 1103, Act of October 3, 1917, authorized April 29, 1922

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NEW SERIES, NO. 158 (VOL. XXII) JUNE, 1927

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WEALTH DISTRIBUTION IN THE CONTINENTAL UNITED STATES AT THE CLOSE OF 1921[1]

By Willford I. King

To measure the income of the people of the United States is anything but an easy task, but to estimate the extent of their wealth is even more difficult. The total of the current income is, to a large extent, made up of the value of gratifications to be delivered in the near future. On the other hand, the wealth total is an aggregate of property values, and most economists agree that the value of any piece of property is equal to the sum of the discounted values of all the services which it is anticipated that the specific piece of property will render in the future. Clearly, then, every value placed upon wealth is based upon guesses concerning the future—not only the guess of the present owner of the property but also of a large number of other persons who might be interested in obtaining it.

It goes without saying that our expectations concerning the future are subject to striking and radical changes in very short periods of time. The fact is not so generally recognized, however, that the necessary corollary of this proposition is that the dollar value of the wealth of the people of the United States is subject to violent fluctuations, these fluctuations being due to the alternate appearance of waves of optimism and waves of pessimism. Everyone is aware of the fact that the value of the securities of a corporation may double in a month even though the physical property belonging to the concern has changed but to a negligible extent. Similarly, during the last two or three years, instances are not wanting in which the value of land in certain cities or subdivisions of cities has been multiplied several times, though the land itself has been improved in no respect. In certain parts of the United States, the value of farm land per acre practically

[1] Presented at the eighty-eighth annual meeting of the American Statistical Association, St. Louis, December, 1926.

doubled between 1916 and 1920, and then diminished some 50 per cent between 1920 and 1924. Yet, during all this time, the fertility of the land remained approximately the same. What changed was the expectations of the farmers, and others interested in farm lands, concerning the probable future prices of crops.

To calculate changes in the volume of wealth, we need a physical scale rather than a dollar standard, but, since index numbers of wealth which can be used to convert wealth valuations in different years to dollars of constant purchasing power have not yet been constructed, such a measurement is hard to attain. The ideal, of course, would be to measure wealth in terms of units of utility. Unfortunately, however, no one has yet devised a feasible method of attaining this ideal. In the present stage of economic science, therefore, the best that can be done is to estimate the value in terms of money of any given aggregate of individual wealth, or, to state it more accurately, the value of private property in the hands of individuals.

In the process of estimating for the National Bureau of Economic Research the income of the people of the United States, it has been necessary to calculate the value of different types of property held by individuals at various dates. The estimates which are here presented were obtained in connection with a special study on *Gradations of Earnings and Income in 1921*. All the figures given are for the close of that year. The fact should be noted that these estimates have not been presented to the Board of Directors of the Bureau for approval and hence are under no circumstances, to be taken as findings of the National Bureau of Economic Research.

While the wealth is divided into categories having little resemblance to those used by the Bureau of the Census, and while the methods followed in calculating some of the items also differ widely from those employed by that Federal Bureau, it will be observed that the totals differ but little from those arrived at by the makers of the Census of Wealth.

Table I records the results of subdividing in three different ways the estimated wealth aggregate of $281,159,000,000.

TABLE I

I. Wealth on December 31, 1921, Classified According to Its Nature

	Millions
Motor cars:	
On farms	$1,012
Urban	3,299
Miscellaneous consumption goods	42,450

	Millions	
Dwellings occupied by owners:		
Urban	$18,047	
Farm	6,282	
Dwellings leased:		
On tenant farms	3,022	
Urban	26,448	
Other real estate:		
Buildings on farms used for business purposes	6,203	
Farm land	42,031	
Urban, manufacturing, mining, etc.	55,080	
Farm implements, livestock, and crops on hand	10,118	
All other wealth	67,167	
Total		$281,159

II. Wealth on December 31, 1921, Classified on the Basis of Corporate, Public, or Individual Ownership

	Millions	
Corporate securities:		
Stocks	$43,043	
Funded debt	25,080	
Bonds of federal, state, local, and foreign governments	24,787	
All other individual property	188,249	
Total		$281,159

III. Property on December 31, 1921, Classified According to Its Ownership by Farmers or Non-Farmers

	Millions	
Farmers:		
Real estate owned by farmer landlords	$2,579	
Land operated by farm owners	21,503	
Buildings on farms of owners	10,470	
Implements on farms of owners	2,348	
Implements on tenant farms:		
Property of farm owners	45	
Property of tenants	1,028	
Livestock on farms of owners	2,837	
Livestock on tenant farms:		
Property of farm owners	199	
Property of tenants	1,173	
Crops on hand	2,489	
Motor cars used for business	304	
Farm mortgages held by farmers	4,235	
Other investments and business property	6,474	
Motor cars used for pleasure	708	
Clothing, furniture, etc.	5,292	
Total gross wealth of farmers	$61,684	
Indebtedness	9,687	
Net wealth of farmers		$51,997

	Millions	
Non-Farmers:		
Securities	$91,200	
Leased real estate used by merchants	6,765	
Leased urban homes	26,448	
Other leased real estate	7,562	
Owned urban homes	18,047	
Direct or consumption goods other than residences	40,458	
Business equipment, merchandise, cash, and miscellaneous items	38,682	
Total wealth of non-farmers		$229,162
Total		$281,159

Space is lacking here to go into detail concerning the methods used in arriving at the estimates presented in Table I and Chart 1 illustrating the division of wealth according to its character. Suffice it to say that while, presumably, all of the items are more or less erroneous, there seems no reason to believe that any of them depart widely from the truth. The values assigned to real estate are taken primarily from Census estimates, the figures for farm property being based upon the Census of Agriculture and those for other types of real estate upon the Census of Wealth, Debt, and Taxation.

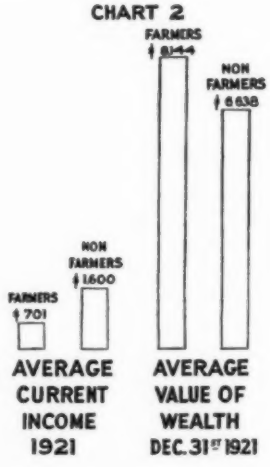
CHART 1

TOTAL INDIVIDUAL WEALTH

DECEMBER 31ST 1921

$ 281,159,000,000

CHART 2



A glance at Chart 1 reveals how important a part is played by real estate, over half of the wealth of the country falling into this category. Further, nearly two-thirds of the nation's wealth is devoted to business as compared to the one-third existing in the form of direct or consumption goods. During the last few years we have heard much of the

poverty of the farmer and, as far as his income is concerned, there is no doubt that these rumors rest on a firm foundation of fact. In 1921 which was a very unprofitable year for the agricultural industry, the average farmer and his family received current income in the form of money and commodities amounting to but $701, while the average gainfully occupied non-farmer obtained income, measured in similar terms, amounting to $1,600—and the latter represents income per person rather than per family. Were the figures for non-farmers put on a per family basis, the discrepancy between the incomes of farmers and non-farmers would be materially increased. On the other hand, 1921 was relatively a worse year for the farmers than for the non-farmers. The figures in Chart 2 are, then, to be taken merely as roughly characteristic of the difference in the income status of farmers and non-farmers.

Clearly, from the standpoint of money income, farmers are decidedly worse off than non-farmers. The case is, however, quite different as regards wealth, the farmers being slightly better equipped in this regard. This fact is illustrated in the right hand section of Chart 2. The explanation is, of course, that agriculture cannot be carried on without land, and, with the growth of population, land has become extremely valuable.

THE DISTRIBUTION OF WEALTH AMONG PROPERTY OWNERS

While the division of wealth into categories is a matter of moment, there is, perhaps, wider interest in the distribution of wealth on the basis of the amount of wealth per person or family. This latter topic has, of course, in the past called forth much discussion, for, from the social standpoint, nothing can be of greater significance. The vital nature of this subject is indicated by the word itself. Wealth means "weal" and "weal" is practically equivalent to "welfare." According to our Constitution, one of the chief purposes in forming our Government was to "promote the general welfare." The accumulation of wealth is the goal toward which the vast majority of mankind is constantly striving, even though most of the wealth acquired is soon used up. However, despite the fact that the urgent wants of the moment are given precedence, most people are not satisfied merely to acquire enough goods to cover their daily needs. They desire, in addition, to accumulate a bank account or a mass of property. This anxiety to get ahead in the world applies to practically all nationalities, races, ranks, and occupations, and a very large proportion of the total time, thought, and energy of the world is expended to this end. True, certain social reformers say that this is all wrong—that the average working man has

no business trying to save but should use up his entire income to further the immediate welfare of his family. However, it is safe to say that these reformers have not yet succeeded in convincing any considerable proportion of the population that their views are sound.

The amount of income is, of course, important. Without income, wealth could never come into existence, for, as the Classical Economists long ago pointed out, all wealth is the result of earning, and saving is the excess of income over consumption. Furthermore, the scale of living of the typical family is dependent primarily upon its current income, though, since a part of current income may be saved, it is perhaps more exact to say that the family's current income determines its potential rather than its actual scale of living.

The concept of income is, nevertheless, very closely tied up with the idea of consumption. If we know that, in a given nation, the average income is high, we also know without inquiring that, in the country mentioned, the people live well and enjoy many of the conveniences of life.

Wealth, on the other hand, may be quite independent of the scale of living, though such is the exception rather than the rule. The outstanding characteristic of wealth is that to its owner it gives power, a term which in this case includes not only the ability to live well, if one wishes, but also the opportunity to control enterprise. As everyone knows, the possession of wealth is a great convenience. It enables one when ill to buy food and medicine, pay hospital fees, and employ the services of a physician, thus facilitating a speedy recovery. When the employee who possesses no wealth loses his position, he must take the first job that offers, go hungry, or become a pauper. On the other hand, the man who has accumulated wealth has time to seek for the type of position for which he is best adapted. The general possession of wealth, then, tends to lessen the number of square pegs finding their way into round holes, thus preventing, of course, a large amount of unhappiness. Not only does wealth help a man to locate a suitable position, but it gives him day by day a feeling of security not enjoyed by the man who owns but little in the way of worldly goods. The family that has accumulated a bank account has a better chance of sending its children to college, thus giving to the members of the younger generation a better opportunity of satisfying their tastes and ideals, and to the parents a vicarious pleasure in observing the success and happiness of their offspring.

Whether wealth is widely distributed or instead concentrated in the hands of the few is a matter of great significance from a political standpoint. Those who possess the wealth of a nation are, oftentimes, its

real rulers. The fact is generally recognized that the country in which most of the wealth is in the hands of a few, while the great majority are propertyless, is one in which it is easy to incite the inhabitants to revolution, for, under such circumstances, the masses feel that they have little to lose through any political upheaval that may occur. On the other hand, in the nation in which the great majority of the inhabitants are property owners, governments tend to be unusually stable; for, since property rights are commonly prized next to life itself, the owners of wealth oppose any policy which threatens the security of their holdings.

METHODS OF MEASURING THE DISTRIBUTION OF WEALTH AMONG FAMILIES

Since the way in which wealth is divided among the inhabitants is a matter of such importance, it is not surprising that many attempts have been made to measure its distribution. The method of attacking the problem commonly employed in the past, has been to study the distribution of the estates of decedents, the records usually being obtained either from the Probate Courts, or from the authorities in charge of the inheritance tax. Until recently, the only well known collections of data of this type pertaining to the United States have been the figures gathered by Carroll D. Wright in Massachusetts covering the four periods 1829—31, 1859—61, 1879—81, and 1889—91, and the material from six Wisconsin counties compiled by Dr. Max O. Lorenz. A most valuable addition to these sources of information has appeared in Chapter 3 of the report of the Federal Trade Commission on *National Wealth and Income*. The Commission has collected data for a total of 43,512 estates—the records being obtained from the County Probate Courts of 12 widely scattered states and the District of Columbia.

The fact is generally recognized that the distribution of estates is not the same thing as the distribution of wealth among the living inhabitants of a nation. Casual observation seems to indicate that the average man in the United States accumulates property and, hence, in his old age, possesses much more wealth than he owned when a boy. We would expect, therefore, to find the average estate probated decidedly greater in value than the average amount of wealth possessed by the individual during life. It may be remarked at this point, however, that, on their face, the figures of the Federal Trade Commission do not seem to bear out this hypothesis.

To say that the distribution of estates is different from the distribution of wealth among the living is far from equivalent to saying that data concerning the former are not worthy of careful study. One

reason, perhaps, that statisticians have tended to study the distribution of estates rather than the distribution of wealth among the living is that the latter concept is not easy to define. One of the difficulties involved has to do with the question of how much wealth is to be assigned to children. While most children have some possessions which they call their own, the majority of them hold little or no property. In portraying distribution, it is evident, however, that we are scarcely justified in picturing the child population as a poverty-stricken section of the inhabitants. Similarly a considerable proportion of the women of the country, from the property standpoint, occupy the same status as children, but it is clearly illogical to class the wife of a rich man as a virtual pauper because she does not happen to possess any property in her own name. For such reasons, it seems evident that wealth distribution among the living should be treated rather on the family than on the individual basis. Unfortunately, however, available statistics give us no adequate picture of the numerical relationships existing between families and individuals or between family wealth and individual wealth. Under such circumstances, the best that can be done is to attempt to measure the distribution of wealth among those who happen to hold title to it, ignoring entirely the fact that others are dependent for support upon the wealth of such property owners.

Even were it possible to measure the distribution of wealth among all living individuals and families with a high degree of precision, it is doubtful that the picture would be as interesting as one showing the distribution of estates among decedents. In the United States there is a widespread belief that "equality of opportunity" should be the birthright of every American. To most of us this phrase conveys largely the idea of equality of opportunity to acquire wealth. The average man's picture of an ideal social organization would probably be one in which every young man, though starting in with nothing except good health and intelligence, could be assured that, by industry and diligence, he would be certain of attaining a competence before he reached old age. Whether we endorse or oppose this thesis, most of us are much interested in knowing how many, under the present organization of society, succeed in attaining the goal described. Beyond question, then, the distribution of wealth prevailing among people who have reached the close of life, is a subject well worthy of careful study.

While popular writers on the subject have often assumed that studies showing the distribution of estates are also typical of the distribution of wealth among the living, it is not believed that any serious student of the subject has fallen into this error. However, the assumption made in the past by most investigators in this field (including the

present writer) has been that the records of estates probated enable us to obtain a satisfactory measure of the distribution of wealth as it exists among persons near the end of their careers. Recently, however, this view has been seriously challenged, chief among the critics being Judge R. S. Galer of Mount Pleasant, Iowa, and Dr. Walter R. Ingalls of New York City. The burden of their criticisms is typified by the argument of Dr. Ingalls in Chapter X of his book *Current Economic Affairs*, in which he cites data collected from many reliable sources which seem to indicate that the poorest 65 per cent of the population own a much larger proportion of the wealth of the nation than the 2 per cent which the distribution of estates in Massachusetts and Wisconsin indicates to be the share of this large majority. How can we explain the apparently contradictory results shown by these analyses of different masses of data supposed to be equally reliable?

The clue to the solution seems to lie in facts pointed out by Judge Galer, namely, that many estates are not probated at all, that some of the real estate—the most valuable type of national wealth—is held by joint title of husband and wife, thus requiring no court action at the death of either party, and that much other real estate is transferred at death without any record being made of the value thereof. It follows then, that records of estates probated may fall far short of showing the entire wealth of the decedents at the time of demise. One might add that, since heavy inheritance taxes have become the rule, there is the strongest incentive for persons anticipating death to arrange means to transfer property to their heirs by devices which will enable the bequests to escape taxation.

How does the hypothesis that the value of estates probated is materially less than the wealth of decedents shortly before death fit in with the statistics appearing in the report of the Federal Trade Commission? An analysis of their figures may help to answer the question. They estimate that, in the counties studied, during the period covered, 259,908 persons died, leaving estates valued at $707,815,744 —an average of $2,727 per person. While we have Census estimates of the total wealth of the nation for the years 1912 and 1922 only, we can, by taking the mean of these two figures, obtain a rough estimate of the average aggregate of wealth during the period 1912 to 1923. By dividing such an estimated total by the average population, we obtain a quotient of $2,453, supposedly representing the approximate average wealth of each living inhabitant during the period covered by the Federal Trade Commission's study. Since, as previously stated, there is reason to believe that the average American accumulates wealth throughout life, we would expect the average wealth among the

decedents to be materially higher than the average wealth of the living. The figures just cited, however, indicate no material difference. Does this not justify one in inferring that much wealth escapes probate?

Apparently, then, there are strong reasons for believing that, in the study of wealth distribution, it is necessary to deal with three rather than with two distributions. The three are as follows:

1. Distribution of estates.
2. Distribution of wealth among persons shortly before their death.
3. Distribution of wealth among all inhabitants living in the area covered.

Existing data throw light on 1 and 3, but nothing bearing on 2 has been discovered. This situation is unfortunate in view of the fact that it may well be nearly as erroneous to confuse 1 and 2 as it is to confuse 1 and 3.

In order to see what the new data collected by the Federal Trade Commission show, I have recast their figures in a form which makes them comparable with the analysis and distribution of current income which it is expected will appear later in a publication of the National Bureau of Economic Research. The procedure adopted is indicated by the following statements: Figures in *Statistics of Income*[1] for 1917 (the year occupying about the middle of the period covered by the study of the Federal Trade Commission) show that the number of *all* income taxpayers was 1.06355 times as great as the number of *male* income taxpayers. The *Mortality Statistics* of the Census of 1921, page 298, indicate that, of all decedents over 21 years of age, 51.949 per cent were males. Because of lack of more definite information, I have assumed that all the wealth of the people of the United States belonged to the 40,508,324 persons either paying income tax or reported in the Census as "gainfully occupied." This assumption is, of course, admittedly faulty, but it is not believed that it does serious violence to the facts. Following out these premises, it has been calculated that, in the 24 counties covered by the Federal Trade Commission's study, 102,195 wealth owners died during the period 1912 to 1923, of whom 43,512 had estates which were probated, leaving 58,683 for whom no estates were reported. In order to make the resulting figures comparable with those appearing in the analysis of estates in Massachusetts and Wisconsin recorded in my book on *The Wealth and Income of the People of the United States*, I have made the same assumption here as there—namely that the non-reported estates were too small in value to be worth reporting. This is the same assumption that has been made by the Federal Trade Commission. On the basis of this hypothesis, I have

[1] Publication of the United States Bureau of Internal Revenue.

TABLE II

THE DISTRIBUTION OF ESTATES OF PROPERTY OWNERS DYING IN THE UNITED STATES DURING THE YEARS 1912 TO 1923 INCLUSIVE*

Estimated on the Basis of the Assumption that the Estates Not Probated Averaged But $100 Each in Value

Value of estate	Number of estates in class	Percentage of total number of estates		Total value of estates in class (thousands)	Percentage of total value of estates		Average value of estates in class
		In class	Above lower limit of class		In class	Above lower limit of class	
Not Probated	58,683	57.423	100.000	$5,868	.87	100.00	$100**
Probated							
Under - $500	6,099	5.968	42.577	1,575	.23	99.13	258
$500 - 1,000	4,824	4.720	36.609	3,388	.50	98.90	702
1,000 - 2,500	8,766	8.578	31.888	14,196	2.10	98.40	1,619
2,500 - 5,000	7,572	7.409	23.311	26,934	3.98	96.30	3,557
5,000 - 10,000	6,446	6.308	15.901	45,161	6.67	92.33	7,006
10,000 - 25,000	5,518	5.400	9.594	85,234	12.59	85.66	15,447
25,000 - 50,000	2,231	2.183	4.194	77,930	11.51	73.07	34,931
50,000 - 100,000	1,105	1.081	2.011	76,040	11.23	61.56	68,814
100,000 - 250,000	651	0.637	0.930	97,600	14.41	50.34	149,923
250,000 - 500,000	179	0.175	0.293	60,326	8.91	35.92	337,016
500,000- 1,000,000	76	0.074	0.117	52,027	7.68	27.01	684,566
$1,000,000 and over	44	0.043	0.043	130,913	19.33	19.33	2,975,295
TOTAL - -	102,195	100.000	100.000	$677,191	100.00	100.00	$662,646

*Estimates based upon data taken from the United States Federal Trade Commission report on *National Wealth and Income*, Table 10.
**A guess.

arbitrarily assigned a value of $100 each to the 58,683 non-reported estates.

The figures based upon these assumptions appear in Table II. They show the total value of the 102,195 estates to have been $677,190,966. The Lorenz curves based upon this table and presented in Chart 3 show that the distribution indicated is roughly the same as that shown by Massachusetts estates at the various periods during the last century for which records exist and by Wisconsin estates in 1900. Apparently, then, the remarkable tendency of probate records at different times and places to have similar distributions is borne out by this new and inclusive collection of data.

For reasons mentioned above, however, it does not seem safe to assume that the distribution of estates as recorded gives a true picture of anything more than the actual distribution of wealth at the time of death and perhaps not even that. It is possible that it comes far from correctly portraying the distribution of wealth among persons shortly before death. It is quite improbable that it gives anything like a true picture of the distribution of wealth among living persons.

THE DISTRIBUTION OF WEALTH AMONG LIVING PERSONS

Dr. Walter R. Ingalls' previously mentioned book on *Current Economic Affairs* indicates a possible method of measuring the distribution of wealth among living persons, which appears to be not only entirely original but also well worthy of pursuing further than Dr. Ingalls has done. This further pursuit has been attempted by the present writer. The rather involved methods here employed give results nominally much more complete than those set forth in *Current Economic Affairs*. There is, of course, a possibility that the present mathematical treatment has been more elaborate than the quality of the material warrants. My belief is that such is not the case, but with this view the reader, when he finishes this paper, may be inclined to disagree.

The attack on the problem was begun by attempting to ascertain the distribution of wealth among farmers.

THE DISTRIBUTION OF WEALTH AMONG FARMERS

According to figures presented earlier in this paper, the farmers of the Continental United States possessed, at the close of 1921, property having a total net value of $51,997,000,000. Our first problem, in apportioning the wealth total, was to estimate the distribution of this amount among the individual wealth classes. Fortunately, the *Census of Agriculture* for 1920, Volume V, page 88, presents figures showing the value of various types of farm property on different sizes of farms. There is little reason to doubt that the wealth of the farmer varies, on the average, with the size of his farm. It has, then, been found possible, by applying rather complex methods to the Census data, to make a rough estimate of the value of the holdings of each class of farm owners and of each class of tenant farmers. Though, without question, estimates made on this basis are necessarily crude, they are, nevertheless, highly significant, for they at least place correctly in their broad general positions on the wealth scale, values amounting to over one-sixth of the wealth of the nation. We know, for example, that the great bulk of this $51,997,000,000 was in the hands of persons possessing less than $100,000 each, and this knowledge helps us greatly in arriving at definite conclusions concerning the degree to which wealth is concentrated in the hands of the very rich.

THE DISTRIBUTION OF WEALTH AMONG NON-FARMERS

Dr. Ingalls' most important contribution to the problem of estimating the distribution of wealth among living persons consists of his pointing out the usefulness for that purpose of the Federal Income Tax data showing the different sources of individual income, data not available

when my study on *The Wealth and Income of the People of the United States* was prepared. In the present study, I have followed Dr. Ingalls' lead in acting on the hypothesis that the value of the wealth of any class of persons is a multiple of the income from wealth received by that class.

THE DISTRIBUTION OF SECURITY HOLDINGS AMONG NON-FARMERS

A careful study of a large number of corporations operating in various fields indicates that, at the end of 1921, the market value of their stocks averaged 15.955 times the amount of dividends paid in 1921. In the report on *Statistics of Income*, issued by the United States Bureau of Internal Revenue, figures are given showing the amount of dividends received by each class of income recipients. Since an analysis of these figures shows that the average amount of dividends per person declines rather regularly as the individual income diminishes, and since we know the dividend total for 1921, it is not difficult to estimate the amount of dividends received by gainfully occupied persons below the income tax limit. There is no reason for believing that the ratio of stock values to dividends is correlated with the incomes of the stockholders, hence it appears that by multiplying the dividends in each income class by 15.955, the $43,043,070,000 representing the approximate market value of the stocks of all corporations in the hands of individuals at the close of 1921, can be approximately distributed among the various income classes.

Similarly, a rather comprehensive study shows that, on December 31, 1921, individuals owned bonds and other evidences of funded debt of corporations having a market value of $25,080,483,000 and, in addition, possessed bonds of various governmental or municipal organizations valued on the market at $24,786,918,000. A study made by the Federal Trade Commission on the subject of *Taxation and Tax Exempt Income* throws light upon the distribution of tax exempt bonds among various income classes. By utilizing the data from this study in connection with those in the columns of Table 6 of *Statistics of Income* for 1921 entitled respectively "Interest and Investment Income" and "Interest on Government Obligations Not Wholly Exempt from Tax," the $49,867,401,000 has been apportioned among the several income classes. This apportionment, though necessarily considerably less accurate than the distribution of the value of corporation stocks, is still anchored firmly on fact.

Of the $92,910,471 in stocks and funded debt distributed by the methods just described, such evidence as is available indicates that but $1,709,794,000 belonged to farmers, leaving $91,200,677,000 in the

hands of non-farmers. Figures presented earlier in this paper, however, show that non-farmers possessed total wealth valued on the market at $229,162,000,000. The remainder awaiting distribution amounted then to $137,962,000,000. Of this total, $40,775,000,000 consisted of leased real estate.

THE DISTRIBUTION OF LEASED REAL ESTATE AMONG NON-FARMERS

On page 54 of the report of the United States Bureau of Internal Revenue for 1921 entitled *Statistics of Income*, there appear three columns, namely: "Profits from sales of real estate, stocks, bonds, etc.," "Rents and royalties," and "Interest and investment income." In computing the distribution of income, it was necessary to estimate the items in these columns for the lower income classes. The method of making this estimate consisted first in finding the average per person in each reporting class in a given column and second in extending the curve obtained by plotting these averages in order to cover persons having incomes below the exemption level. By multiplying the averages thus arrived at by the estimated numbers of persons in the corresponding income classes, products were obtained which presumably represent the approximate income derived from the specified source by the people in the various income classes below $2,000.

After the figures for each of the columns mentioned had been estimated by the process described, the figures for the three columns were combined for each income class. Since it was felt that the items in the column entitled "Profits from sales of real estate, stocks, bonds, etc." might indicate fairly well the extent of holdings of real estate in the various classes, and since the total amount of such income as reported to the Bureau of Internal Revenue was relatively small, it seemed desirable to weight the items in this column by doubling the items. It appeared, however, that the column entitled "Rents and royalties" constituted by far the best gauge for the distribution of holdings of leased real estate among the various income classes; hence the items in this column were multipled by seven. The figures in the column labeled "Interest and investment income" were taken as they stand to represent the share of the mortgage owners; the reason for including this category in the make-up of these weighted sums being that a considerable equity in leased real estate belonged to money lenders, and their share in the income would be received in the form of interest rather than rent. The weighted sums thus obtained were used as relatives on the basis of which to calculate the share of each income class in the value of real estate leased by merchants, the value of leased urban homes, and the value of miscellaneous leased urban real estate.

As stated above, the total value of the leased real estate apportioned to the various income classes on the basis just mentioned, was $40,775,000,000. The allocation of this amount left to be divided among the different income classes $97,187,000,000, or approximately three-sevenths of the estimated total wealth of non-farmers, the total being valued at $229,162,000,000. With four-sevenths of the wealth of non-farmers pigeon-holed in various income classes with at least a fair degree of accuracy, the next problem was to discover some way of dealing with the remaining three-sevenths (or $97,187,000,000), in such a way as to give an approximately correct picture of existing relationships.

THE DISTRIBUTION AMONG INCOME CLASSES OF DIRECT GOODS, AND MISCELLANEOUS PROPERTY OWNED BY NON-FARMERS

The first item under this head to be distributed consisted of the $18,047,000,000 worth of urban homes occupied by their owners, combined with the $40,458,000,000 worth of other direct or consumption goods in the hands of non-farmers, making an aggregate of $58,505,000,000. In its study of the value of consumption goods possessed by individuals, the Bureau of the Census found that there was rather close correlation between the value of the stock of such goods owned and the income of the family possessing them. In view of this fact, it seemed reasonable to assume that the $58,505,000,000 worth of direct goods calculated to be in the possession of non-farmers at the end of 1921 were distributed among this group approximately in proportion to the distribution of current money income among the same section of the population. The total value of the two types of wealth just mentioned was then distributed in proportion to the "entire current money income" totals for the corresponding income classes as indicated by an unpublished study of the National Bureau of Economic Research.

All of the wealth of non-farmers was now distributed, except the $38,682,000,000 representing the value of miscellaneous items such as business and office equipment, merchandise on hand, and cash. These kinds of property, in so far as they were possessed by individuals, were doubtless owned in part by entrepreneurs. One's first thought would be that their holdings of wealth of this type might well be apportioned among the various income classes in about the same way as the income of the corresponding classes derived from "Business" and from "Partnerships, fiduciaries, etc." as reported in *Statistics of Income* for 1921, page 54. However, in constructing the actual index used for the apportionment, both these columns were weighted no more heavily

than the one entitled "Profits from sales of real estate, stocks, bonds, etc."—this course being justified by the probability that a large proportion of the income under these heads represented rewards to entrepreneurs for their efforts, and would therefore be more akin to wages and salaries than to rewards for the services of wealth. It was felt indeed, that the holdings of business equipment would be fully as likely to vary in proportion to the size of the items in the last mentioned column, hence they were also included in making up the index, and since, in terms of absolute numbers, these items were small, they were multiplied by ten in order to give them approximately the same weight as the larger items in the two preceding columns. The figures in all three columns were estimated for income classes under $2,000 by the process briefly described above in connection with the extension of the curve representing "Rents and royalties." The $38,682,000,000 was then divided among the income classes in proportion to the sizes of the weighted totals obtained by combining the items in the three columns.

THE DISTRIBUTION OF WEALTH AMONG WEALTH CLASSES

By the processes described above, estimates were arrived at showing the distribution of the wealth of non-farmers among the various income classes. The next process was to convert the income classes into wealth classes. This was done by utilizing a rather complicated method which cannot be described here because of lack of space, but which will probably be dealt with later in a report of the National Bureau of Economic Research.

The next step was to add together for each class the number of farmers and the number of non-farmers. Similarly the wealth of farmers

CHART 3

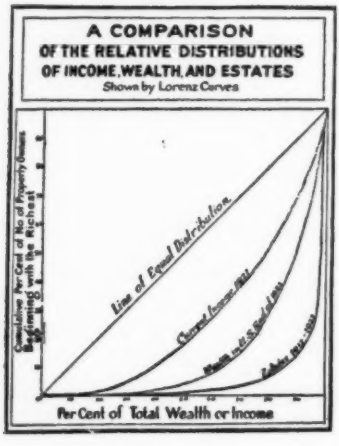
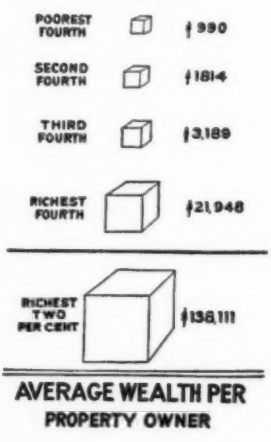

A COMPARISON
OF THE RELATIVE DISTRIBUTIONS
OF INCOME, WEALTH, AND ESTATES
Shown by Lorenz Curves
Line of Equal Distribution
Per Cent of Total Wealth or Income

CHART 4



was combined with that of the non-farmers in the corresponding class. The resulting estimates appear in Table III and are illustrated in Charts 3 and 4.

It would be folly to contend that the figures in Table III are possessed of the degree of accuracy indicated by the number of places to which they have been carried, for they are based upon numerous assumptions, some of which are doubtless very faulty, and upon mechanical methods of adjustment which are far from being error proof. Nevertheless, it is believed that, in a broad way, the figures do indicate the distribution of wealth among the property owners living in the United States at the close of 1921. Since that date wealth has increased largely, but in view of the fact that over a period of nearly a century, great stability has been shown in the relative distribution of estates, the chances are that the relative distribution in 1921 differs but little from the relative distribution in 1926.

A glance at the Lorenz curves in Chart 3 tells us that while the distribution of wealth among living persons was less equal than the distribution of income among the same group, it was decidedly more equal than the distribution of estates. The poorest half of the property owners apparently possessed only about 4 per cent of the wealth though they received some 19 per cent of the income. However, the aggregate value of the poorest half of the estates studied amounted to but 1 per cent of the total.

Half of the income of the country was received by the most prosperous 22 per cent of recipients. Half of the wealth belonged to the richest tenth of the property owners, while half of the total values of estates was included in the property of less than 1 per cent of the wealthiest decedents.

Chart 4 gives us a picture of the average wealth of each quarter of the population arrayed according to wealth, in comparison to the wealth of the richest 2 per cent of the pr perty owners. In picturing these facts, the use of cubes has been resorted to, in order to make the wealth of the poorest fourth visible on the chart. Clearly the distribution is far from equal!

Whether great inequality is desirable or undesirable doubtless will continue to remain a much mooted question. Many will feel it a crime that poverty is allowed to persist in the presence of the wanton waste inevitable to a certain extent as long as individuals have absolute control over great fortunes. Others will hold that, until the birth-rate of the poor is further restricted, the poverty of certain classes is an evil from which there can be no escape. Some will see in great wealth the negation of democracy. Others will picture it as the force which spurs

TABLE III

THE ESTIMATED DISTRIBUTION OF WEALTH AMONG PROPERTY OWNERS IN THE CONTINENTAL UNITED STATES, DECEMBER 31, 1921

Wealth per person	Simple distribution		Cumulative distribution			
			Numbers or amounts above lower limit of class		Per cents above lower limit of class	
	Number of persons in class	Wealth of class (millions)	Number of persons	Amount of wealth (millions)	Number of persons	Amount of wealth
$0 up to $200	113,751	$ 14	40,908,324	$281,159	100.00	100.000
200 " " 400	552,641	175	40,794,573	281,145	99.72	99.995
400 " " 600	1,132,180	578	40,241,932	280,970	96.37	99.933
600 " " 800	1,702,744	1,200	39,109,752	280,392	95.60	99.727
800 " " 1,000	1,787,062	1,615	37,407,008	279,192	91.44	99.30
1,000 " " 1,200	2,007,961	2,221	35,619,946	277,576	87.07	98.73
1,200 " " 1,400	2,522,341	3,293	33,611,985	275,355	82.16	97.94
1,400 " " 1,600	2,896,127	4,354	31,089,644	272,062	76.00	96.76
1,600 " " 1,800	2,672,287	4,528	28,193,517	267,708	68.92	95.22
1,800 " " 2,000	2,037,972	3,861	25,521,230	263,180	62.39	93.61
2,000 " " 2,200	1,873,951	3,921	23,483,258	259,320	57.40	92.23
2,200 " " 2,400	1,655,142	3,795	21,609,307	255,398	52.82	90.84
2,400 " " 2,600	1,456,342	3,631	19,954,165	251,603	48.78	89.49
2,600 " " 2,800	1,306,407	3,518	18,497,823	247,973	45.22	88.20
2,800 " " 3,000	1,136,486	3,288	17,191,416	244,455	42.02	86.95
3,000 " " 3,400	2,064,210	6,587	16,054,930	241,166	39.25	85.78
3,400 " " 3,800	1,639,432	6,000	13,990,720	234,579	34.20	83.43
3,800 " " 4,200	1,432,072	5,706	12,151,288	227,979	29.70	81.09
4,200 " " 4,600	1,040,357	4,563	10,719,216	222,273	26.20	79.06
4,600 " " 5,000	791,795	3,791	9,678,859	217,710	23.66	77.43
Thousands of Dollars						
5 up to 6	1,560,250	8,521	8,887,064	213,919	21.72	76.06
6 " " 7	1,075,500	6,961	7,326,814	205,398	17.91	73.05
7 " " 8	813,800	6,062	6,251,314	198,437	15.28	70.58
8 " " 9	576,200	4,876	5,437,514	192,375	13.29	68.42
9 " " 10	472,000	4,469	4,861,314	187,499	11.88	66.69
10 " " 11	400,300	4,188	4,389,314	183,029	10.73	65.10
11 " " 12	322,150	3,690	3,989,014	178,841	9.75	63.51
12 " " 13	252,200	3,143	3,666,864	175,152	8.96	62.30
13 " " 14	216,450	2,914	3,414,664	172,009	8.35	61.18
14 " " 15	192,860	2,789	3,198,214	169,095	7.82	60.14
15 " " 16	177,130	2,739	3,005,354	166,306	7.35	59.15
16 " " 17	162,870	2,681	2,828,224	163,567	6.91	58.18
17 " " 18	148,630	2,595	2,665,354	160,885	6.52	57.22
18 " " 19	140,180	2,588	2,516,724	158,290	6.15	56.30
19 " " 20	127,340	2,478	2,376,544	155,702	5.81	55.38
20 " " 25	494,550	11,035	2,249,204	153,224	5.50	54.50
25 " " 30	427,450	11,675	1,754,654	142,188	4.29	50.57
30 " " 40	510,900	17,512	1,327,204	130,514	3.24	46.42
40 " " 50	186,000	8,268	816,304	113,001	2.00	40.19
50 " " 100	371,100	25,619	630,304	104,734	1.54	37.25
100 " " 500	230,520	46,844	259,204	79,115	0.63	28.14
500 " " 1,000	21,450	14,439	28,684	32,271	0.070	11.48
1,000 " " 2,500	5,800	7,518	7,234	17,832	0.018	6.34
2,500 " " 5,000	785	2,779	1,434	10,314	0.0035	3.67
5,000 " $10,000	345	2,224	649	7,536	0.0016	2.68
10,000 " $20,000	269	3,680	304	5,312	0.00074	1.89
20,000 " $40,000	28	738	35	1,631	0.00009	0.58
40,000 and over	7	893	7	893	0.00002	0.32
All Amounts	40,908,324	281,159				

men on to greater effort thus making for national efficiency, in addition to giving real zest to life. Some will emphasize the wasteful practices of the idle rich who squander time and talents which should be usefully employed in productive enterprise. Others will contend that, without great concentration of wealth in the hands of the few, the corporations responsible for the production of such a large proportion of our national income would never have continuing policies and hence would necessarily become inefficient and unproductive—a result which would bring disaster to employer, employees, and consumers alike. However, the purpose of this study is not to decide these issues but merely to present such estimates of the distribution of wealth among the living as existing materials make possible.

THE SIZE OF AMERICAN FAMILIES IN THE EIGHTEENTH CENTURY*

AND THE SIGNIFICANCE OF THE EMPIRICAL CONSTANTS IN THE PEARL-REED LAW OF POPULATION GROWTH

By Alfred J. Lotka, *Statistical Bureau, Metropolitan Life Insurance Company*

The Pearl-Reed law of population growth can be written

$$r = r_0\left(1 - \frac{N}{N_\infty}\right) \tag{1}$$

where r is the rate of increase per head when the population is N, while r_0 is the *incipient* rate of increase, that is to say, the rate which would prevail in the initial stage of growth from a very small nucleus. The symbol N_∞ denotes the *ultimate* population, the upper limit of possible growth.

Expressed in words formula (1) states that the rate of increase at any time is equal to the *incipient* rate of increase diminished by a fraction expressing the existing population in terms of the ultimate.

For the United States equation (1) in numerical form is

$$r = .03134\left(1 - \frac{N}{197{,}273{,}000}\right). \tag{2}$$

Thus, the American population in its initial career, according to the law of Pearl and Reed, as quoted in numerical form above in equation (2), should have increased at the *incipient* rate of 3.134 per cent per annum. We have, of course, no record of this *incipient* rate of increase, but using the earliest figures available, we have for 1790

$$\left.\begin{aligned} \frac{N}{N_\infty} &= \frac{3{,}929{,}000}{197{,}273{,}000} = .01991. \\ \left(1 - \frac{N}{N_\infty}\right) &= .98009. \end{aligned}\right\} \tag{3}$$

The population in 1790 was thus about one-fiftieth of the ultimate, and the departure from the *incipient* state is not very great. In fact, calcu-

*This paper forms a sequel to an article published in the September, 1925, issue of this Journal, on "The True Rate of Natural Increase," and the writer wishes here to express his cordial thanks to Dr. Dublin for the opportunity which has been given him thus to continue an inquiry begun in collaboration with him.

lating r by the Pearl-Reed formula we find for the year 1790, on the basis of $N = 3{,}929{,}000$,

$$r = .0307$$

as against $r = .0301$ observed 1790–1800

and $r_0 = .03134$ (*incipient* rate).

The observed increase was not wholly due to excess of births over deaths. Immigration during the period 1790–1820 has been estimated at about 250,000,[1] of which we may assign about 75,000 to the decade 1790–1800. This would give for the natural rate of increase (excess of birth-rate per head over death-rate per head)[2]

$$r = .0287.$$

These figures—the *incipient* rate of increase of our population, and its close approach in actual early observations—are of intrinsic interest as characteristics of our growth curve, and it appears worth while to look a little more closely into their implications. Interpreted in terms of fertility, what does this *incipient* rate of increase of 3.134 per cent per annum represent? What does this figure tell us regarding such matters of common interest as birth-rate, size of family, maternity frequency among women of reproductive ages? What, in this sense, is the upper limiting fertility of American women, that which set the limit to population growth at a time when population pressure was as yet wholly unfelt?

We find some assistance in our approach to the problem thus raised, by the fact that in the early stages the Pearl-Reed curve corresponds very nearly to growth in geometric progression. In these circumstances the total births from year to year also increase in geometric progression at the rate r. Then, if T is the mean length of one generation, and R the ratio of total births in two successive generations, we find, applying the principle of continuous compound interest for T years

$$R = e^{rT}. \tag{4}$$

Let us, then, cast our problem first of all in the following form. Denoting by R_0 the ratio of total births in two successive generations according to conditions of mortality and maternity frequency in 1920; by how much would we have to multiply the maternity frequencies of 1920 to obtain a value of r such as observed in the early history of our

[1] C. S. Burr, *America's Race Heritage*. 1922, pp. 272, 274, 275.

[2] It is interesting to compare this figure with the estimate of Malthus that a population increasing without constraint doubles itself in 25 years. This corresponds to an annual rate of increase per head of .0277.

population? How many times greater than today was the fecundity of the women at the time of the incipient rate of increase of 3.134 per cent per annum?

Let k be the required multiplying factor. Then, in place of (4) we have

$$kR_0 = e^{rT}. \tag{5}$$

More exact discussion will follow. As a preliminary we can very easily obtain a rough approximation, in applying (5) numerically, as follows:

The mean length of one generation T (from mother to daughter) is nearly independent of k and r and of the particular life table prevailing;[1] its value is about 28 years.[2] The value of R_0 in 1920 was 1.168. Thus we have

$$1.168k = e^{28r}$$

$$k = \frac{e^{28r}}{1.168}.$$

With the *incipient* rate of increase r_0 we have, therefore, since $28r_0 = .878$

$$k = \frac{e^{.878}}{1.168} = 2.06.$$

That is to say, the fecundity of the women in the *incipient* period of our growth must have been just about twice as great as today. For the year 1790, with $r = .0287$, we find similarly

$$k = \frac{e^{.804}}{1.168} = 1.91.$$

It adds a concrete touch to the exposition to express these facts in another way, namely, by inquiring how many children per female born and per married woman would issue in each of the cases considered.

The number of *daughters* per female born is, of course, equal to the ratio of total births in two successive generations, kR_0, that is to say 2.41 for the *incipient* rate of increase, and 2.23 for the conditions of 1790. The corresponding number of *children* per female is obtained by multiplying by 2.06, since 1.06 sons are born for every 1.00 daughters. Thus we find for the *incipient* rate of increase 4.96 children per female, and for 1790, 4.60 *children*. But only a fraction of

[1] As shown later, p. 164.

[2] This JOURNAL, Vol. 20, 1925, p. 334.

all females born marry. In 1920, as shown in Table I, this fraction was .782;[1] for 1790, with a higher mortality, and, therefore, fewer females

TABLE I

FEMALES DYING SINGLE AT SEVERAL AGES AND AT ALL AGES ACCORDING TO SEVERAL SCHEDULES OF MARITAL RATIO AND MORTALITY, IN LIFE TABLE COHORT OF 100,000 WHITE FEMALES

Basis of computation							
Mortality as of	1920	....	1920	....	1838–54	1838–54	1838–54
Marital ratio as of	....	1920	1920	1790	....	1790 Augmented	1920
(1) Age	(2) Life table deaths d_x 1920	(3) Marital ratio % single* 1920	(4) Deaths single 1920 (2) × (3)	(5) Marital ratio % single* 1790	(6) Life table deaths d_x 1838–54	(7) Deaths single* 1790** (5) × (6)	(8) Deaths single* 1790† (3) × (6)
0–15	11,950	100.0	11,950	100.0	30,308	30,308	30,308
15	240	98.6	237	98.0	387	379	382
16	270	95.6	258	91.6	416	381	398
17	302	88.1	266	81.6	452	369	398
18	340	80.5	274	71.5	492	352	396
19	377	71.2	268	61.0	534	326	380
20	410	61.9	254	51.4	577	297	357
21	440	54.0	238	43.7	587	257	317
22	465	47.5	221	37.3	597	223	284
23	486	40.7	198	31.8	605	192	246
24	505	34.9	176	27.2	612	166	214
25	520	30.8	160	23.3	619	144	191
26	532	27.2	145	20.0	626	125	170
27	543	24.0	130	17.1	632	108	152
28	550	21.2	117	15.3	637	97	135
29	555	19.1	106	13.6	643	87	123
30	560	17.9	100	11.9	648	77	116
31	563	16.3	92	10.8	653	71	106
32	564	15.4	87	9.7	657	64	101
33	563	14.6	82	8.9	662	59	97
34	562	14.0	79	8.3	667	55	93
35–44	5,650	11.9	672	6.9	6,951	480	827
45–54	7,800	10.0	780	6.8	7,707	524	771
55–64	12,800	8.7	1,114	6.7	10,917	731	950
65–∞	52,600	7.3	3,840	6.6	32,417	2,140	2,366
			21,844			38,012	39,878
1 − .2184 = .7816 deaths married.‡				1 − .3801 = .6199 deaths married.‡			1 − .3988 = .6012 deaths married.‡
1/.7816 = 1.279 daughters per married white female.				1/.619 = 1.613 daughters per married white female.			1/.6012 = 1.663 daughters per married white female.

* *Single* here means *never married* and excludes widows and divorced.
** Life table 1838–54 combined with marital ratio 1790 estimated as indicated in Chart I.
† Life table 1838–54 combined with marital ratio of 1920.
‡ Married here means "ever married" and includes widowed and divorced. For convenience this figure is here stated per unit, not per 100,000 females born.

[1] This recomputed figure differs slightly in the third decimal from that given (.785) in a previous publication, this JOURNAL, 1925, p. 327. In Table VI, p. 326, of that publication the figure is given as .748 through an accidental transposition of digits, which, however, is not carried into subsequent computations.

surviving to reproduce, an estimate explained below gives a figure of .620. On the basis of this latter figure we find for the number of children per married woman, in the *incipient* period 8.0 children, and for 1790, 7.4 children. Finally, if we assume, for these early days, that seven per cent of all marriages were sterile,[1] we find for the *incipient* period 8.6 children, and for 1790, 8.0 children *per mother*.

The approximate method of computation given above is remarkable chiefly for the ease with which numerical results are obtained thereby. It remains now to establish these results on a more rigorous basis. In doing this we shall at the same time be enabled to draw conclusions regarding the effect of certain detail influences to which we have so far paid no attention.

More Exact Treatment

It has already been pointed out that in the early stages the Pearl-Reed curve corresponds to growth in geometric progression. This, in the presence of only negligible amounts of immigration and emigration, implies a nearly fixed age distribution of the form [2]

$$c(a) = be^{-ra}p(a) \tag{6}$$

where $c(a)$ is the coefficient of age distribution, such that the proportion of persons between ages a and $a+da$ is given by $c(a)\,da$. The symbol $p(a)$ denotes the probability, at birth, of reaching age a, and b is the birth-rate per head. The formula (6) holds separately for each sex, and it will be convenient in what follows to restrict our attention to females.

The value of the proportional rate of increase r is determined by the relation [3]

$$1 = \int_0^\infty e^{-ra}\,\mu(a)\,m'(a)\,p(a)\,da \tag{7}$$

where $\mu(a)$ is the proportion of females married at age a, and $m'(a)$ is the maternity frequency among the married females,[4] that is, the number of children (daughters, since we restrict our considerations to females) born per annum per married woman of age a.

[1] Karl Pearson, in *Chances of Death*, 1897, Vol. I, p. 69, estimates that involuntary sterility occurs in about 6 to 8 per cent of all marriages. It is here assumed that in 1790 there was little or none of the modern birth control, or similar conditions.

[2] Sharpe and Lotka, *Phil. Mag.*, April, 1911, p. 435; Lotka, *Elements of Physical Biology*, pp. 110, 113, 118, 1925.

[3] A. J. Lotka, *loc. cit.*

[4] The symbol $m'(a)$ is here used to denote the maternity frequency referred to the *married* female population (of age a), as distinguished from $m(a)$, the maternity frequency referred to the total female population (of age a); the latter symbol and concept only were employed in the prior publication cited, and this note is here added to prevent misunderstanding.

Numerical Application

We have, of course, no life table that should furnish us with values of $p(a)$ for the period 1790–1800. The nearest reliable approach to such a table we find in the English Life Table for the period 1838–1854, and this will accordingly be used in the ensuing computations. The factor $\mu(a)$ also is unknown for the period under consideration. However, this quantity is not subject to very wide variations. Computations will be carried out on two different bases, namely, (1) using values of $\mu(a)$ as observed in 1920, and as set forth in Table I, column 3; (2) using *augmented* values obtained by arbitrarily raising the 1920 figures as indicated in Table I, column 5, to allow for the greater frequency of marriages at the early period considered. It will be found that this modification does not very greatly affect the result of the computations to be presented.

The method employed to make allowance for the "modified" or "augmented" marital ratio can be gathered from Table II. So, for example, the maternity frequency, per 100,000 females of all marital states, in the age group 15–19, estimated for 1790, is obtained by dividing the corresponding figure for 1920, namely, 2202, by the percentage married in 1920, namely, 11.5 and multiplying by the percentage married in 1790, namely, 18.0; thus $\frac{2202}{11.5} \times 18.0 = 3447$.

TABLE II

AGE SCHEDULE OF MATERNITY FREQUENCY (DAUGHTERS ONLY) FOR "WHITE FEMALES" AND "MARRIED WHITE FEMALES" 1920 (OBSERVED) AND 1790 (COMPUTED)

(1) Age	(2) Maternity frequency per 100,000 females of all marital states, 1920 $m(a)$	(3) Per cent actually* married, 1920 $\mu_1(a)$	(4) Per cent actually* married "augmented"† for 1790 $\mu_2(a)$	(5) = (2)/(3) Maternity frequency per 100,000 married females, 1920 $m'(a)$	(6) = (5) × (4)/100 Maternity frequency per 100,000 females of all marital states, calculated, 1790 $\mu_2(a)m'(a)/100$
10–14	9	?	?	?	(9)
15–19	2,202	11.5	18.0	19,148	3,447
20–24	7,310	50.8	61.0	14,390	8,778
25–29	7,480	73.1	80.0	10,233	8,186
30–34	5,780	80.3	86.0	7,198	6,190
35–39	3,898	81.4	87.0	4,789	4,166
40–44	1,552	80.0	84.0	1,940	1,630
45–49	172	76.8	80.0	224	179
50–54	5	72.0	75.0	69	5

* Excluding widowed and divorced.

† Estimated as in Chart I, with an allowance for *widowed and divorced* based on the difference between Table I, column (3), and the complement of Table II, column (3), which difference corresponds to the "widowed and divorced" as observed in 1920.

We shall, on this basis, inquire into the character of the maternity frequency $m'(a)$ which must have prevailed to yield the observed value of $r = .0287$ in 1790; and we shall similarly determine the upper limiting maternity frequency corresponding to the *unrestrained* or *incipient* rate of increase of 3.134 per cent, characteristic of our population, according to the formula of Pearl and Reed.

TABLE III

FACTOR k BY WHICH 1920 FECUNDITY WOULD HAVE TO BE MULTIPLIED TO GIVE SEVERAL RATES OF INCREASE r UNDER SEVERAL STATED CONDITIONS ALSO, CONSTANTS EMPLOYED IN COMPUTATION

Basis of computation			
Mortality as of.....	1920	1838–54 (English Life Table)	1838–54 (English Life Table)
Age schedule of maternity frequency as of.....	1920	1920	1920
Marital ratio as of.....	1920	1920	1920 "modified" to represent 1790
r	k		
−.030.....	0.357	0.475	.426
−.025.....	0.416	0.552	.488
−.020.....	0.482	0.640	.560
−.015.....	0.557	0.740	.646
−.010.....	0.644	0.855	.746
−.005.....	0.744	0.988	.856
0.000.....	0.856	1.137	.985
+.005.....	0.985	1.308	1.129
.010.....	1.135	1.507	1.296
.015.....	1.305	1.727	1.483
.020.....	1.500	1.987	1.698
.025.....	1.721	2.274	1.942
.02866.....	1.899	2.511	2.138
.030.....	1.970	2.605	2.215
.03134.....	2.045	2.699	2.296
R_0.....	1.168	0.880	1.015
R_1.....	33.23	24.88	28.11
R_2.....	998.79	743.36	825.38
R_3.....	31,581	23,363	25,599
R_4.....	1,045,342	768,839	834,122
α.....	28.45	28.29	27.69
β.....	−45.71	−44.77	−46.28
γ.....	54.32	59.46	68.83
δ.....	141.03	160.44	171.13

The problem is indeterminate unless we adopt some convention regarding the nature of the change in the form of the function $m'(a)$. The simplest convention is that the contour of the maternity frequency curve or polygon is to remain geometrically similar,[1] $m'(a)$ being simply

[1] Aside from its simplicity, this assumption finds some support also from what is known as "Tait's Law" of linear fertility decrease with age. See G. U. Yule, *Journal Royal Statistical Society*, 1906, p. 115.

CHART I

VALUES OF COEFFICIENT k (ORDINATES), BY WHICH 1920 OBSERVED MATERNITY FREQUENCIES MUST BE MULTIPLIED TO PRODUCE GIVEN VALUES OF "ULTIMATE" RATE OF INCREASE r (ABSCISSAE)

The symbol 1838/1790 indicates that basis of computation was English Life Table 1838–1854, marital ratio as of 1790 (estimated). Similarly for other like symbols.

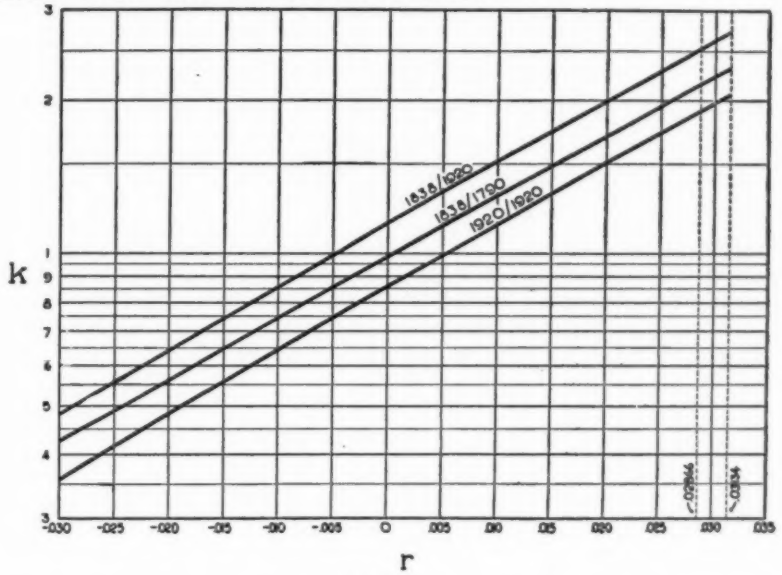
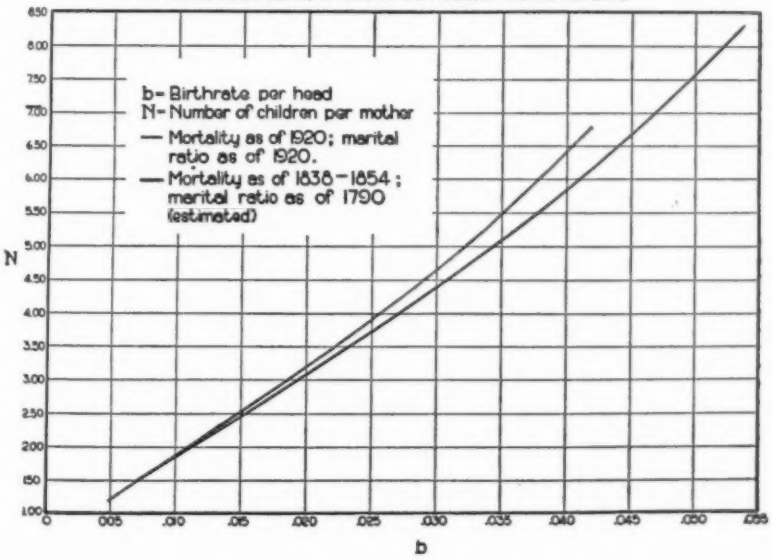


CHART II

RELATION BETWEEN BIRTH-RATE PER HEAD b AND NUMBER OF CHILDREN N PER MOTHER, UNDER SEVERAL CONDITIONS


b= Birthrate per head
N= Number of children per mother
— Mortality as of 1920; marital ratio as of 1920.
— Mortality as of 1838–1854; marital ratio as of 1790 (estimated)
N
b

transformed into $km'(a)$, where k is a factor independent of a. In this case (7) becomes

$$1 = k\int_0^\infty e^{-ra}\,\mu(a)\,m'(a)\,p(a)\,da \qquad (8)$$

an equation which enables us to compute k for every value of r when $\mu(a)$, $m'(a)$ and $p(a)$ are given. Thus it is found that for $r = .0287$, representing the state of affairs in the United States 1790, $k = 2.1$, on the basis of the *augmented* 1790 marital ratios. This means that the maternity rates at the date mentioned must have been well over twice those of the present day. The corresponding factor k for the *incipient* rate of increase of 3.134 per cent is 2.30. If, instead of the augmented marital ratios, we use the ratios as of 1920 direct, we find $k = 2.5$ and 2.7 respectively for $r = .0287$ as of 1790, and for the *incipient* $r = .03134$. For further details see Table III and Charts I, III, and IV. In reading the table and the graph it must be noted that the table gives

CHART III

HISTOGRAM OF MATERNITY FREQUENCY (COUNTING SONS AND DAUGHTERS) PER 1,000 MARRIED WHITE FEMALES

(a) As observed in 1920, that is, with $k = 1$. (b) Augmented to give *incipient* rate of increase $r = 3.134$ per cent; $k = 2.045$. (c) Reduced to give $r = 0$ (stationary population); $k = 0.856$.

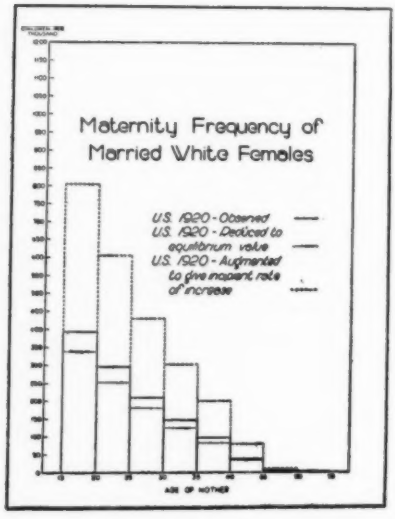
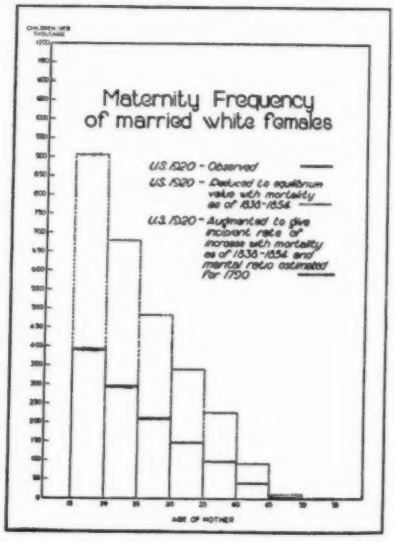

Maternity Frequency of Married White Females
U.S. 1920 - Observed
U.S. 1920 - Reduced to equilibrium value
U.S. 1920 - Augmented to give incipient rate of increase
AGE OF MOTHER

CHART IV

HISTOGRAM OF MATERNITY FREQUENCY (SONS AND DAUGHTERS) PER 1,000 MARRIED WHITE FEMALES

(a) Same as in Chart III. (b) With mortality according to English Life Table 1838–1854, and maternity frequency augmented, $k = 2.296$ to give *incipient* rate of increase $r = 3.134$ per cent. (c) Same as (b), but with maternity frequency reduced, $k = 0.985$, to give stationary population.


Maternity Frequency of married white females
U.S. 1920 - Observed
U.S. 1920 - Reduced to equilibrium value with mortality as of 1838-1854
U.S. 1920 - Augmented to give incipient rate of increase with mortality as of 1838-1854 and marital ratio estimated for 1790
AGE OF MOTHER

daughters only in the "maternity frequency" column, while the graphs are drawn for "sons and daughters," that is, 2.06 times the figures for daughters only.

The same facts find somewhat more telling expression if we compute the number of children born per married woman, corresponding to the conditions indicated. It is desirable first of all, however, to effect a transformation. Equation (8) is not convenient when a series of values of k are to be computed. It is then better to write it in the form [1]

$$kR_0 = e^{r(\alpha + \frac{1}{2}\beta r + \frac{1}{3}\gamma r^2 + \frac{1}{4}\delta r^3 + \ \ldots)} \tag{9}$$

where
$$R_n = \int_0^\infty a^n \mu(a)\ m'(a)\ p(a)\ da \tag{10}$$

and
$$\left.\begin{aligned} \alpha &= \frac{R_1}{R_0} \\ \beta &= \alpha^2 - \frac{R_2}{R_0} \\ \gamma &= \alpha^3 - \frac{3}{2}\alpha\frac{R_2}{R_0} + \frac{1}{2}\frac{R_3}{R_0} \\ \delta &= \alpha^4 - 2\alpha^2\frac{R_2}{R_0} + \frac{2}{3}\alpha\frac{R_3}{R_0} + \frac{1}{2}\left(\frac{R_2}{R_0}\right)^2 - \frac{1}{6}\frac{R_4}{R_0}. \end{aligned}\right\} \tag{11}$$

Now kR_0 is the ratio of the total births of daughters in two successive generations, or, in other words, it is the number of daughters born, on an average, to every female born. If M is the fraction out of all women born, who eventually marry, then $\frac{kR_0}{M}$ is the number of daughters born, on an average, per married female. And since 1.06 sons are born for every 1.00 daughters, the number of children born per married woman is $\frac{2.06kR_0}{M}$. The fraction M is easily determined as in Table I when the proportion of females in the single state and the death-rate are known for each age of life.

Applying these principles it is found that in the conditions represented by the populations of the United States in 1790 and on the assumptions indicated above, the number of children born, on an average, per fertile [2] married woman, was 7.76, on the basis of the *augmented* marital ratio. The average number of children corresponding to the

[1] Essentially as set forth in this JOURNAL, Vol. 20, 1925, p. 332.

[2] In each case allowance is made for 7 per cent sterility of marriages.

incipient rate of increase is, on the same basis, 8.33. This then would represent the upper limiting fertility of American women in the eighteenth century. It is interesting to compare these figures with that given by Whetham [1] as the result of a study of 100 families of the British nobility 1831–1840, namely, 7.1 children.[2]

The results cited above, together with a number of other related quantities, are collected together in Table IV which is self-explanatory.

Mean Length of Generation

The expression

$$\alpha+\tfrac{1}{2}\beta r+\frac{1}{3}\gamma r^2+\ \ldots\ =T \tag{12}$$

which occurs in the exponent of (9) measures the mean length T of one generation from mother to daughter, as has been shown on a previous occasion.[3] Hence, in first approximation

$$T=\alpha=\frac{R_1}{R_0}. \tag{13}$$

Now the integrals R_1, R_0 both contain $p(a)$ as a factor of the integrand. In consequence of this the influence of the particular life table used in forming the quotient $\frac{R_1}{R_0}$ tends very largely to cancel out, and, as has been remarked before, the mean length of one generation T is accordingly nearly independent of the particular life table prevailing. It is also approximately independent of r because the coefficient α is numerically considerably greater than $\frac{1}{2}\beta r$ for such values of r as enter into consideration. It is in consequence of this that the figures in columns (2) and (5) of Table IV are almost identical, although they are based upon widely different life tables. In concrete terms this means that the number of children *per female* required to yield a given rate of increase per head r is essentially independent of the age schedule of mortality (life table). Where the saving of an improved life table shows up is in the number of children *per married female* or *per mother*, because, with a favorable life table, a larger proportion of females survive to marry, and

[1] W. C. D. and G. D. Whetham, *The Family and the Nation*, p. 139, 1909; quoted A. Newsholme, *Vital Statistics*, p. 105, 1923.

[2] Comparison should also be made with the figures obtained by the short method given at the beginning of the present paper. These were, for 1790, 8.0 children; for the *incipient* rate of increase 8.6 children per mother; as against 7.76 and 8.33 by the more rigorous method.

[3] This JOURNAL, Vol. 20, 1925, p. 334.

therefore the burden placed upon each married female is decreased. This is clear from columns (3), (4), (6), (7), (9) and (10) of Table IV.

TABLE IV

RELATION BETWEEN r RATE OF INCREASE PER HEAD OF POPULATION, AND NUMBER OF CHILDREN PER FEMALE, PER MARRIED FEMALE, AND ALSO PER MOTHER, UNDER SEVERAL CONDITIONS

Basis of computation									
Mortality as of.....	1920			1838–54 (English Life Table)			1838–54 (English Life Table)		
Age schedule of maternity frequency as of.....	1920			1920			1920		
Marital ratio as of ..	1920			1920			1920 "modified" to represent 1790		
r	Number of children per female								
(1)	(2) All females	(3) Married females	(4) Mothers	(5) All females	(6) Married females	(7) Mothers	(8) All females	(9) Married females	(10) Mothers
−.030.....	.86	1.10	1.18	.86	1.43	1.54	.88	1.42	1.52
−.025.....	1.00	1.28	1.38	1.00	1.66	1.78	1.02	1.64	1.76
−.020.....	1.16	1.48	1.59	1.16	1.93	2.08	1.17	1.89	2.03
−.015.....	1.34	1.71	1.84	1.34	2.23	2.40	1.35	2.18	2.35
−.010.....	1.55	1.98	2.13	1.55	2.58	2.77	1.56	2.51	2.70
−.005.....	1.79	2.29	2.46	1.79	2.98	3.20	1.79	2.89	3.11
.000.....	2.06	2.64	2.84	2.06	3.43	3.69	2.06	3.32	3.57
+.005.....	2.37	3.03	3.26	2.37	3.94	4.24	2.36	3.81	4.10
.010.....	2.73	3.49	3.75	2.73	4.54	4.88	2.71	4.37	4.70
.015.....	3.14	4.02	4.32	3.13	5.21	5.60	3.10	5.01	5.39
.020.....	3.61	4.62	4.97	3.60	5.99	6.44	3.55	5.73	6.16
.025.....	4.14	5.30	5.70	4.12	6.85	7.37	4.06	6.55	7.04
.02866.....	4.57	5.85	6.29	4.55	7.57	8.14	4.47	7.22	7.76
.030.....	4.74	6.06	6.52	4.72	7.85	8.44	4.63	7.48	8.04
.03134.....	4.92	6.30	6.77	4.89	8.13	8.74	4.80	7.74	8.33

Birth-rate

The birth-rate corresponding to the *incipient* rate of increase of 3.134 per cent may very properly be calculated on the supposition that the age-distribution is that corresponding to geometric rate of increase, since the early stages of growth follow very nearly that law. The same assumption is also entirely fair for the value $r=.0287$ corresponding to 1790. The death-rate then follows immediately from $b-d=r$. The values thus obtained, on the basis of the English Life Table 1838–1854, were

r	b	d
.03134	.0537	.0224
.02866	.0508	.0221

For other values of r the values of b and d computed on the assump-

tion of geometric increase represent artificial norms rather than figures comparable with those actually observed, especially in the United States, where the influence of immigration has been very marked. However, as a matter of arithmetical interest the values of the birth-rate b and the number of children N per mother corresponding to various values of r are set forth in Table V and Charts II and V.

Interval Between Births

The reciprocal of the maternity frequency at any age evidently gives the average interval between births. This figure is of special interest at the ages of maximum fertility, 15–19 years, for here the extreme limit is set by obvious biological necessity. And, in point of fact, the interval obtained at the *incipient* rate of increase, namely 13¼ months, is clearly related to the extreme limit of frequency of pregnancy.[1] For further details on this point see Table VI.

This throws an interesting light on the biological basis of the exponential constant, .03134 in the Pearl-Reed formula for American population growth.

TABLE V

RELATION BETWEEN BIRTH-RATE PER HEAD b AND NUMBER OF CHILDREN N PER MOTHER* UNDER SEVERAL STATED CONDITIONS OF MORTALITY AND MARITAL RATIO; IN EACH CASE COMPUTED ON THE BASIS OF A FIXED AGE DISTRIBUTION CORRESPONDING TO THESE STATED CONDITIONS AND TO A MATERNITY FREQUENCY k TIMES THAT OBSERVED IN 1920

For values of k corresponding to the values of r in column (1) see Table III

Basis of computation						
Mortality as of.....	1920		1838–54		1838–54	
Marital ratio as of.....	1920		1920		1920 Augmented for 1790	
r	b	N	b	N	b	N
−.030.....	.0048	1.18	.0072	1.54	.0072	1.52
−.025.....	.0061	1.38	.0091	1.78	.0091	1.76
−.020.....	.0078	1.59	.0013	2.08	.0113	2.03
−.015.....	.0097	1.84	.0139	2.40	.0139	2.35
−.010.....	.0119	2.13	.0169	2.77	.0169	2.70
−.005.....	.0145	2.46	.0202	3.20	.0202	3.11
.000.....	.0174	2.84	.0239	3.69	.0239	3.57
*.005.....	.0206	3.26	.0279	4.24	.0279	4.10
.010.....	.0241	3.75	.0322	4.88	.0322	4.70
.015.....	.0279	4.32	.0369	5.60	.0369	5.39
.020.....	.0320	4.97	.0418	6.44	.0418	6.16
.025.....	.0362	5.70	.0469	7.37	.0469	7.04
.02866.....	.0394	6.29	.0508	8.14	.0508	7.76
.030.....	.0406	6.52	.0522	8.44	.0522	8.04
.03134.....	.0418	6.77	.0537	8.74	.0537	8.33

* Allowing 7 per cent sterility in marriages.

[1] Carr Saunders, *The Population Problem*, cites 11 months as the actual limiting interval. This, however, does not represent an *average*. The figure given above, 13¼ months, is the *average* interval under the conditions stated.

TABLE VI

AVERAGE INTERVALS BETWEEN BIRTHS AT SEVERAL AGES OF MOTHERS ACCORDING TO SEVERAL STATED SCHEDULES OF MATERNITY FREQUENCY COMBINED WITH STATED SCHEDULES OF MORTALITY AND CORRESPONDING RATES OF INCREASE r OF THE POPULATION

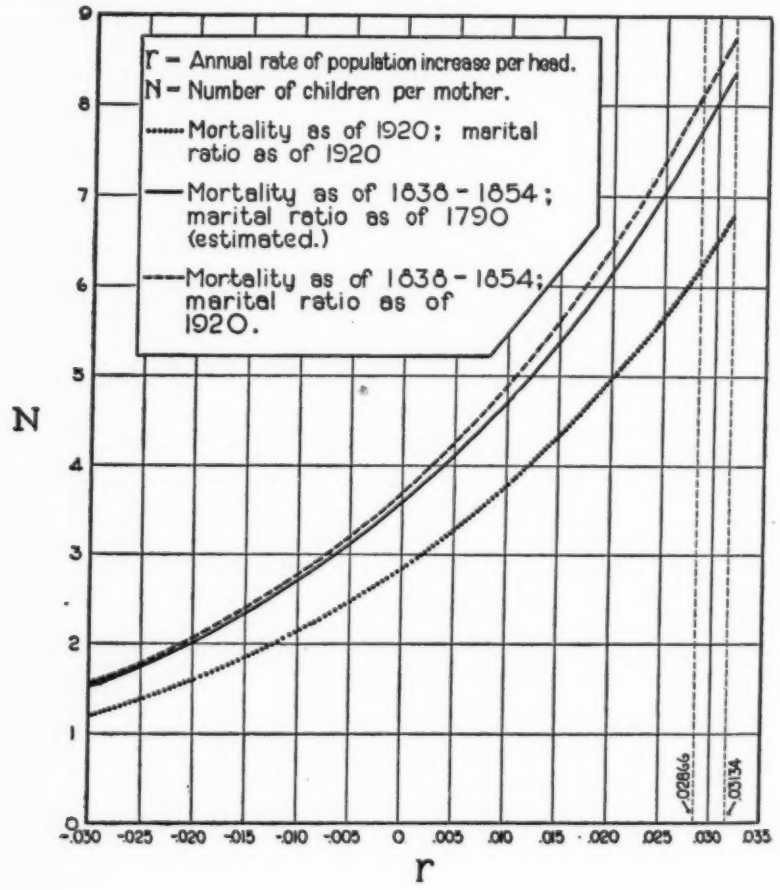
Basis of computation													
Mortality as of.....			1920		1920			1838–54			1838–54		
Marital ratio as of.....			1920		1920			1920			1920 Augmented for 1790		
Age	Maternity frequency per 100,000 married females per annum, 1920 observed												
	Daughters	Children	r = .00547*	r = .00547*	r = 0	r = .02866	r = .03134	r = 0	r = .02866	r = .03134	r = 0	r = .02866	r = .03134
			Years	Months									
10–14													
15–19	19,148	39,445	2.54	30.5	35.62	16.06	14.92	26.83	12.15	11.30	30.96	14.27	13.28
20–24	14,390	29,643	3.37	40.4	47.19	21.27	19.76	35.53	16.09	14.96	41.00	18.90	17.60
25–29	10,233	21,080	4.74	56.9	66.46	29.96	27.83	50.05	22.66	21.08	57.75	26.61	24.78
30–34	7,198	14,828	6.74	68.9	80.48	36.28	33.69	60.60	27.44	25.53	69.93	32.23	30.01
35–39	4,789	9,865	10.14	121.7	142.15	64.07	59.52	107.04	48.46	45.09	123.52	56.92	53.01
				Years									
40–44	1,940	3,996	25.03	....	29.24	13.18	12.24	22.02	9.97	9.28	25.40	11.71	10.90
45–49	224	461	216.9	....	253.4	114.2	106.1	190.8	86.4	80.4	220.1	101.5	94.5
50–54	6.9	14	7,143.	....	8,368.	3,761.	3,493.	6,283.	2,844.	2,647.	7,249.	3,341.	3,111.

NOTE. — At the higher ages the average interval between births is evidently meaningless as applied to an individual mother. Its significance here is to show the extreme rarity of maternity at these ages.

* Ultimate rate of natural increase, corresponding to 1920 age schedule of mortality and maternity frequency. See Dublin and Lotka, "On the True Rate of Natural Increase," this JOURNAL, September, 1925, p. 333.

CHART V

RELATION BETWEEN NUMBER OF CHILDREN *N* PER MOTHER AND RATE OF ANNUAL INCREASE *r* PER HEAD OF POPULATION UNDER SEVERAL CONDITIONS


r = Annual rate of population increase per head.
N = Number of children per mother.
Mortality as of 1920; marital ratio as of 1920
Mortality as of 1838 - 1854; marital ratio as of 1790 (estimated.)
Mortality as of 1838 - 1854; marital ratio as of 1920.
N
r
.02866
.03134
-.030 -.025 -.020 -.015 -.010 -.005 0 .005 .010 .015 .020 .025 .030 .035
0 1 2 3 4 5 6 7 8 9

Second Limiting Factor in Pearl-Reed Law

The second limiting factor in the Pearl-Reed law, which finds expression in the constant N_∞, may be conceived as primarily of economic character. The limit of population that the country can support evidently depends on agricultural and trade relations. Nevertheless, this factor also has its biological expression in the maternity frequency, under given conditions of mortality, that just provides population equilibrium. The factor k_0 by which 1920 maternity rates must be multiplied to give the equilibrium maternity (in conjunction with the 1920 mortality schedule) is .86. Thus the 1920 mortality schedule provides, in this sense, a 14 per cent margin of safety on the plus side of equilibrium. If in the course of time our mortality rates are further improved, the margin of safety will be correspondingly increased.

SUMMARY

The mean length of one generation is not greatly affected by changes in the life table or in the age schedule of fecundity. Its value is about 28 years. The annual rate of increase of the American population about the end of the eighteenth century was just about 3 per cent per annum. It follows that the proportion of total births in two successive generations at that time (resulting at a rate of continuous compounding of interest at 3 per cent for 28 years) was 2.3. This corresponds to a family of 7.8 children per married woman, or 8.3 children per mother, the latter figure computed on the assumption that 7 per cent of marriages are naturally sterile.

A particular interest is added to the significance of these figures when they are considered in the light of the Pearl-Reed law of population growth. It is then found that the exponential constant .03134 which appears in the Pearl-Reed formula is of the nature of a biological constant, characteristic of the American population. It represents the *incipient* or *unfettered* rate of increase such as corresponds to the free expansion of a small nucleus of population in a wide territory before the pressure of the boundaries becomes felt. The biological nature of this upper limit of the rate of increase is seen when the fertility of women corresponding to this rate of increase is expressed in terms of the average interval between births. It is when found that in the age group 15–19, where the fertility is at its maximum, the average interval between births corresponding to the limiting rate of increase is 13¼ months. Evidently, we strike here the fundamental limiting factor of human population increase.

This upper limit of unfettered reproductive power is shown to exceed

the fecundity as of 1920 in the ratio of about two to one. The lower limit of fecundity corresponding to an ultimate state of stationary population under present conditions of mortality, is .86 of the present (1920); that is to say, present conditions of fecundity allow a 14 per cent margin above the stationary condition. This latter statement is based on the 1920 life table. Improvement in mortality rates will in some degree increase the margin indicated.

Incidentally, a relation is set forth between the number of children per mother, the birth-rate, and the rate of increase of the population, corresponding to a given age schedule of mortality and of maternity frequency.

PRICE INDEX OF THE NATIONAL BANK OF COMMERCE IN NEW YORK

By E. M. Miller and F. J. Cavanaugh

All business activity centers in price. For this reason, price fluctuations are justly regarded as perhaps the most sensitive of all indications of business trends. But obviously all prices are not of equal value in this regard, because the many classes of goods traded in have varying relations to the intricate structure which we call business. There is considerable difference in the promptness with which the various types of wholesale prices respond to commercial and financial weather signals. As a general proposition, it may be said that raw materials are the most sensitive, while the prices of finished goods as they pass into the hands of retailers or other final distributors are the least so.

The violent price fluctuations of 1919 and 1920 afforded unusually clear examples of this lag from the raw material to the finished product. Thus, according to the United States Bureau of Labor Statistics, the peak price for calfskins and goatskins was reached in August, 1919. Immediately thereafter a decline began. Chrome calf leather reached its high in November and glazed kid in December, 1919, and quotations were unchanged from these levels until April and May, 1920, respectively, when they began to decline. Shoes, however, did not reach their highest wholesale quotations until early in 1920 and prices did not break until June of that year. It is obvious that the effect of the combination of these various components into a single index number is to lessen its sensitivity.

The index here presented, which is now being published monthly in *Commerce Monthly*, issued by the National Bank of Commerce in New York, has been devised with one purpose in view—to construct an indicator which will respond with the utmost promptness and sensitiveness to the varying fortunes of business. It is not intended to picture the general price level. It is designed to show in what direction those prices most closely related to business activity are moving. These requirements have determined the component elements and mechanism of the index.

COMMODITIES AND SOURCES OF QUOTATIONS

In contrast to the all-inclusive lists of indexes designed to show the general level of prices this index is necessarily restricted to a brief and rigorously selected list. It is limited in the first place to raw materials

TABLE I
WHOLESALE PRICE INDEX OF THE NATIONAL BANK OF COMMERCE IN NEW YORK, 1919 TO 1927
(Three-year average 1923–25 equals 100)

	January	February	March	April	May	June
1919	128.7	121.4	120.6	122.9	128.0	137.4
1920	167.7	161.0	161.4	162.1	155.6	150.6
1921	87.9	80.8	76.4	73.0	75.7	71.6
1922	76.8	76.6	77.3	79.8	83.1	87.3
1923	99.6	102.4	105.6	105.4	101.5	99.1
1924	93.0	95.2	93.6	90.6	88.3	87.1
1925	110.9	109.3	107.3	101.8	102.3	104.8
1926	105.8	103.1	98.6	96.3	94.8	95.8
1927	91.5	89.6	88.8	87.1		

	July	August	September	October	November	December
1919	151.7	156.4	150.3	151.2	161.5	162.3
1920	147.4	141.3	132.4	118.8	103.7	90.2
1921	70.4	69.1	71.0	73.3	74.0	75.9
1922	89.4	88.3	90.6	97.0	97.9	96.8
1923	92.9	92.4	94.1	95.2	92.0	91.4
1924	89.6	95.4	96.4	99.4	105.7	107.2
1925	107.8	109.4	109.1	108.0	110.2	105.7
1926	97.9	97.2	98.0	97.2	94.4	93.3

TABLE II
WHOLESALE PRICE INDEX OF UNITED STATES BUREAU OF LABOR STATISTICS CONVERTED TO 1923–25 BASE

	January	February	March	April	May	June
1919	128.9	125.4	127.0	128.8	131.1	131.5
1920	151.2	150.7	152.0	158.6	159.9	157.7
1921	110.1	103.8	100.8	95.9	94.3	91.8
1922	89.7	91.7	92.2	92.5	95.7	97.0
1923	101.0	101.6	102.8	102.9	101.3	99.5
1924	98.0	98.4	97.2	96.2	95.2	93.8
1925	103.7	104.1	104.4	101.3	100.6	102.1
1926	101.1	100.5	98.2	98.0	98.4	98.7
1927	95.2	94.9	94.2			

	July	August	September	October	November	December
1919	137.5	140.0	136.3	137.0	140.8	144.8
1920	156.1	150.0	146.7	137.0	127.3	116.2
1921	91.4	91.7	91.7	91.8	91.2	90.6
1922	100.4	100.5	99.4	99.9	100.8	101.3
1923	97.6	97.3	99.7	99.3	98.6	97.9
1924	95.3	97.1	96.5	98.5	99.0	102.2
1925	103.7	104.0	103.5	102.2	102.2	101.3
1926	97.7	96.7	97.6	97.1	96.0	95.4

either exactly as won from nature or in the first stage for which a real market exists. Thus raw sugar, steel and pig lead have been processed, but there is no genuine trading in the raw materials, sugar cane, iron and lead ore, from which they are derived.

Mere restriction to raw materials would not be sufficient for the purpose of this index, however; for the prices of some raw materials, particularly those with highly specialized uses or produced in few places,

hence easily controlled, move tardily and often are more or less conventionalized. But most of the more basic raw materials—commodities which enter heavily into many lines of consumption—meet the desired requirements. They are widely and actively traded in, many of them on organized exchanges; they feel economic pressure quickly and information as to their prices is definitely and speedily available.

The list of commodities chosen as a basis for this index perhaps leaves something to be desired. In one case it was necessary to use the price of a finished product, burlap, in place of its raw material, jute, because little jute comes to the United States in the raw form and market prices are therefore nominal for a large part of the year. Save for steel beams included in the *Iron Age* composite for finished steel, no commodity entering directly into the building industry is included, an admitted weakness. This is due to the fact that thus far it has proved impracticable to secure accurate spot quotations from the chief markets for building materials. Eventually lumber may be included. The best that can be said for the list finally selected is that it meets, as well as it was found possible to do, the requirements of this index.

No index number is better than the quotations which enter into it. It is true that minor errors which may creep into the prices used in a very broad index such as that of the United States Bureau of Labor Statistics, are of little importance, but in an index such as that under discussion the prices used must be the best that can be secured. The difficulty of getting for those commodities not traded in on an open exchange quotations which can be accepted as fairly reflecting price changes from month to month and from year to year is too little appreciated by those out of actual touch with the markets.

For instance, on a given date a commodity may be just coming out of the doldrums, with a nominal range of prices reported. Whether the quotation used will reflect a rise, a decline or no change will depend on the accuracy of judgment of the person responsible for its selection.

Then there is the distracting problem of grade. Of the list of commodities included in this index, silk is the most conspicuous example of this difficulty. The lack of standards in grading it has thus far rendered impossible the establishment of a futures market in New York. At best, efforts to follow its true price movement over a series of months or years have brought only moderate success.

Another type of difficulty arises from the seasonal factor. The case of cattle and hog prices combines the difficulties of extremely complicated grading and seasonal differences in market desirability both from the standpoint of weight and quality, due in part to changes in the proportion of total deliveries falling into the various classifications at

different times of year and in part to the effects of weather on consumption. There is even a certain weekly cycle in the livestock markets.

Some fifteen grades of hogs are regularly quoted at Chicago. Under ordinary conditions, the market is overloaded with extremely heavy weights of inferior quality during early midsummer, when old sows are being sold off farms, while it is likely to receive too high a proportion of the very light weights in September and early October when farmers are rushing spring pigs out in the endeavor to avoid the downward price trend which develops as they move to market on the full autumn tide. In consequence, very heavy weights are likely to be penalized in early midsummer, very light weights in September and early October. But if lard stocks should for any reason be low in early summer heavy hogs may escape this discrimination, or if lard stocks should be high in early autumn heavy hogs may fail to reach the favorable price relation usually to be expected at that time.

The weekly cycle of the livestock markets arises from loading and shipping practice. It is the custom to utilize the week-end to move hogs and cattle to market. They go on the cars in large numbers Fridays and Saturdays and arrive in the pens at central stockyards on Mondays, Tuesdays and Wednesdays, on which days the market is broad and active. Prices during the latter half of the week are much less representative than for the first half.

To meet these conditions it is necessary to use special methods to get quotations for cattle and hogs if they are to afford an accurate basis of comparison over a long period. In *Commerce Monthly*, since its Wholesale Price Table was begun, and in this index a privately prepared average price of all sales of cattle and hogs within specified descriptions for the first three days of the week nearest the fifteenth of each month is used.

Another type of difficulty is encountered in the case of commodities where most of the business is done on a contract basis. The conspicuous case is that of coal. Obviously contract prices are not quotable. The best that can be done is to use a spot price, though it represents but a small part of the coal movement.

In those cases where the commodities used in this index are traded in on an organized exchange the prices used in computing it are those quoted on such exchanges. Quotations for other commodities are subjected to the judgment and criticism of those constantly in touch with the actual markets, and it is, therefore, believed that they accurately reflect the price situation as of the available date nearest to the fifteenth of each month.

Below are listed the commodities included in this index, with full descriptions of the grades and quotations used.

Burlap, 40 inch, average of 8 and 10½ ounce weights, New York.
Cattle, fair to choice native steers, Chicago.
Coal, bituminous, run of mine, Pittsburgh District, f. o. b. mine.
Coffee, Santos No. 4, New York.
Copper, electrolytic, early delivery, New York.
Corn, No. 2 mixed, Chicago.
Cotton, middling, spot, New Orleans.
Hides, green salted packers, No. 1 heavy native steers, Chicago.
Hogs, good merchantable, pigs and rough stock excluded, Chicago.
Lead, pig, early delivery, New York.
Linseed oil, raw, New York.
Petroleum, crude, at well, Kansas-Oklahoma.
Rosin, grade H, Savannah.
Rubber, plantation, first latex crepe, New York.
Silk, Japan, best No. 1 to extra, New York.
Silver, commercial bars, New York.
Sugar, 96° centrifugal, duty paid, New York.
Steel:
  Heavy melting scrap, Pittsburgh.
  Finished steel—*Iron Age* composite price covering 87 per cent of United States output.
Tin, Straits, New York.
Wheat, average of No. 1 northern spring, Minneapolis, No. 2 hard winter, Kansas City, and No. 2 red winter, Chicago.
Wool, clean basis, average of Ohio fine delaine, Boston, one-half blood territory, Philadelphia, three-eighths blood territory, Philadelphia, and Ohio one-fourth blood, Boston.
Zinc, prime western, early delivery, St. Louis.

METHOD OF COMPUTING THE INDEX

Accurate quotations secured, the next problem is to employ a method which will utilize rather than mask these desirable attributes of the component commodities. There are obvious difficulties in the way of using prices mechanically as indicators of business trends. Clearly, the sharp drop of cotton prices in 1926 under the weight of an abnormal crop showed little or nothing about the demand for that commodity. No more was the sharp rise of coal prices in early 1920 significant. It reflected a railroad strike; but it succeeded in throwing out all of the well-known price indexes into which it entered.

Such influences, which are not significant or are positively distorting from the standpoint of those interested in trend, are continuously cropping up. Since the end of 1919 the price structure has been affected by such things as the coal and steel strikes of that year, the railroad yardmen's strike of 1920, the shopmen's strike of 1922, rubber

prices soaring under the influence of restriction, prices of natural chemicals declining in the face of the rapid commercialization of synthetics, and the recommendations of the Indian Currency Commission which threaten materially to alter the character of the demand for silver. Coffee valorization, steel combines abroad, the influence of government in bolstering prices of silk, sugar or cotton, tariff adjustments and similar items are continually presented for evaluation to the student of current markets. Hardly less troublesome is the opposite tendency of prices for which there is not an open speculative market or exchange, to stay put.

In so far as possible, then, an index of the type here desired must combine great sensitiveness to price change with ability to escape, in some measure at least, the distorting effects both of abnormal fluctuation and immobility. This has been accomplished by employing an averaging process based on a modified median, rather than some form of arithmetic average such as is commonly used in price indexes.

The usual choice of the arithmetic average as a measure of central tendency assumes that the ordinary laws of probability are operating. Variations from the average are expected to be about as many above as below, and variations of a major kind should be very rare. Under conditions similar to those above outlined, however, the price change in the particular commodity affected is likely to be very different from the average movement of the group and at least in a short list of quotations taken on a month-to-month basis there can be little expectation of finding counter-balancing items. The prices in these cases are answering to individual influences, which in any specific market are of much greater moment than the centralizing tendency of the general business trend.

An ordinary method of meeting this difficulty is to multiply the number of price quotations used. In one case at least and with a different purpose in mind the prices of various services have been added to broaden the scope of the general commodity index. But it has already been pointed out that for the business analyst this method may hide the thing he most wishes to determine. For him the prices of many things—say lemons or chisels, opium or the fee of the veterinary —have a very tardy, if any, significance. Price information that is to delineate the rapidly shifting panorama of commercial activity is likely to be valuable only in so far as it reflects business reactions and business sentiment in the principal centers of economic force.

There is another reason for brevity in the price list. The interpretation of a price index is a difficult thing. When the number of quotations is large frequency studies of the changes may clarify the situation

a great deal. But a proper interpretation of price phenomena cannot be made without adequate knowledge of the principal factors affecting the individual markets themselves. Some of the most detailed surveys now made give almost no consideration to this phase of interpretation, but for the purpose here in view the advantages of intensive study seem to outweigh easily those that follow upon the attempt to make the price compilation co-extensive with the commercial fabric itself.

Nor can abnormal price swings always be properly discounted merely by extending the range of quotations. When the railroad strike of the spring of 1920 sent spot prices of bituminous coal soaring to ten times their previous level, it injected a factor making for higher transportation and manufacturing costs on such a wide scale as to impart an artificial element of strength in a very wide range of markets. In effect this extraneous factor was given a manifold weighting. The indexes of the United States Bureau of Labor Statistics and the various groupings of the Federal Reserve Board, for example, pointed to a sharp ascent in the general price level. Yet the experience of the business community will confirm the findings here presented that the weakening of the price structure was already well under way at that time.

To meet this problem it was decided to abandon the use of any form of arithmetic or geometric average in favor of some other measure of central tendency. Had the number of quotations included been large enough the mode might have been used. With a brief list the median appeared best adapted to the purpose in hand. When the distribution of the data approximates the so-called normal, the accuracy of the median is not much less than that of the arithmetic mean.[1] On the other hand, its function in minimizing the influence of the sharp deviations already alluded to determined its selection. The application of more intricate methods founded on individual frequency studies was possible, of course, but simplicity argued in favor of the form adopted.

The advisability of weighting was considered and passed upon negatively. The invisible weighting which is always present and which is most important because it affects the quotations directly has already been commented upon. Almost any system of weights adopted would at times accentuate changes which from the business viewpoint are distortions. Moreover, in the present state of our knowledge it is impossible to define mathematically the significance of a price movement in one market in comparison with a similar or contrariwise movement in another. The relative exchange values of those commodities marketed, which are used in general price-level determination, do not

[1] F. Y. Edgeworth, *Journal of the Royal Statistical Society*, July, 1923, p. 577, and June, 1914, p. 733.

appear to have any direct bearing on the question involved here. The significance which can by hindsight be attributed to the various movements in the past may be a fertile field for study, but the applicability of the results to conditions which may obtain in the future is open to some question.

It was determined, therefore, to work out this index by regarding each item of price information as a separate reflection of the general tendencies at work, rating each one as valuable as the next except in so far as the simple median acts to discount those prices which seem to have wandered far afield. The automatic weighting which occurs in the very choice of the commodities was not, of course, forgotten, and much effort was expended to preserve a proper diversification in the list. In the case of steel two classifications were chosen from the same industrial field to give some recognition to the importance of this commodity in modern life.

DERIVATION OF THE PRICE RELATIVES

The choice between a fixed base method of computation and the chain index form was decided in favor of the latter. To quote Professor Mitchell: "The period chosen as base for the relative prices should be that period with which accurate comparisons are most significant for the purpose in hand."[1] In this case the computation is made once a month using prices on the nearest available date to the fifteenth. Averages for the months were not used partly because it is desirable in this work to represent the condition actually obtaining at the moment. Even if we had preferred them, however, considerations of time and cost would have rendered their use impossible. This index number must go to press not later than the eighteenth of each month.

In the same way the prime point of reference is the thirty-day period which has just elapsed. The percentage changes were therefore calculated on this basis, the median of these changes was taken and these median relatives were chained together to give a continuous picture of the price movement over the period covered.

In the selection of the medians which were to be employed as representative of the price trend each month a departure was made from the commonly employed method. This departure was based in part on the findings which are reported on pages 19 and 20 of *Bulletin 284, United States Bureau of Labor Statistics*, and in part upon the market experience of the statistical division of the National Bank of Commerce in New York. The charts presented in the bulletin mentioned reveal at a glance that in an extended list of commodities the number of prices

[1] *United States Bureau of Labor Statistics, Bulletin No. 284*, p. 82.

which remain unchanged from year to year is out of all proportion to what it might have been expected to be. If this is true when twelve months have elapsed between the dates to which the quotations apply comparison made after an interval of only thirty days would be even less fruitful in determining the direction of current price movements. Casual reference either to Dun's or Bradstreet's weekly lists will show that by far the greater number of items recorded remain stagnant from one week to the next. The use of the median in an ordinary way for such data would, of course, mean that the central item would always be found in the unchanged group and the index would remain permanently fixed at the 100 per cent level.

The reason for this phenomenon in the grouping of the price data both in the United States Bureau of Labor Statistics and in the Dun and Bradstreet lists is to be found in the nature of the commodities included and probably to some extent in the character of the quotations obtained for these commodities. There are, it is true, some markets which do exhibit a sluggish adjustment to the economic forces at work and which seem of too great importance to be excluded from an index of this sort. If, however, quotations were obtainable which did measure the ever changing gradations of tone which make the true market, the absolute number of fixed quotations should be very small.

As an experimental proposition the median in its usual form was applied to the carefully selected list of price quotations used in the present study and the tendency for the index to remain fixed showed itself only at rare intervals. However, as the primary interest in this computation is the direction of movement rather than the absolute level of prices it appeared best to follow the preceding reasoning to its logical conclusion and to eliminate from consideration in any given month those quotations which recorded no change over the previous thirty days. The result is an index which as a rule must move in some direction, and which is determined in an upward direction whenever the number of price advances is greater than the number of those which have declined, and vice versa.

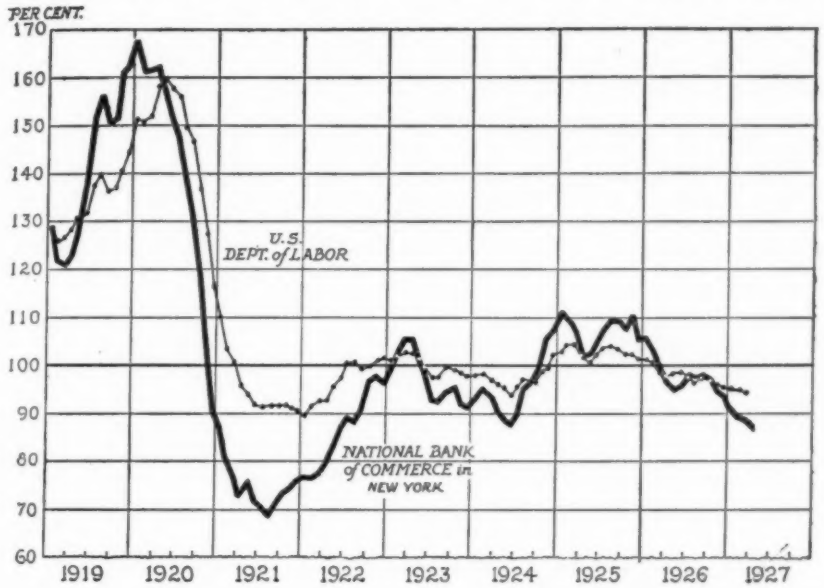
BASE PERIOD

Instead of following the almost standard custom of referring price series to 1913 or to some other pre-war period as a base, the three years from 1923 to 1925 have been used for several reasons. It would be unwise to make comparisons as far back as 1913 by means of an index based on so narrow a list of commodities because of important changes which have taken place in conditions surrounding the production and use of a number of them. Thus, the current widespread use of rosin

for the paper, soap, paint and varnish and linoleum industries lends an interest to the price of this commodity which it did not have some years ago. Lowered costs of production in the case of copper and some other nonferrous metals are quite likely altering the relationship of the prices of this group to the general price structure. The development of the automotive industry, of rubber production by plantation methods and the interjection of artificial restriction of output into the situation have developed in the rubber market conditions which would make comparisons with rubber prices in 1913 of no value whatsoever. In the silk market, rayon has developed as a control factor.

The question immediately arises as to how similar changes in the future will affect the validity of the period 1923–25 as a base if they vitiate comparisons on a 1913 base. The answer is obvious. That such shifts in conditions surrounding major commodity markets will occur in the future, as they have in the past, is beyond all question. Thus, silver prices hitherto so significant of business currents in the Far East are apparently now about to lose some of this interest and a new interpretation may be indicated for this factor. There is only one way to avoid distortion of this index number as a result of the operation of such influences. Changes will have to be made from time to time in the list of commodities used, as some lose significance and others attain it. A price index designed to link up closely with general business must be a living, not a fixed, automatic thing. In a chain series, if and when changes appear wise, they may be made without alteration of the work already done.

With the use of a pre-war base eliminated the three years from 1923 to 1925 appeared to be a logical period, since by 1923 recovery had been made from the market debacle of 1920–22, and since the index of the United States Bureau of Labor Statistics indicates that during the three years used the general price level was fairly stable. It will require the passage of years to determine whether or not the base chosen is a good one. The main reservation on this point in the minds of those who have worked upon it is the effect on the course of world prices and hence on United States prices of the deflation of currency and credits going on in many countries in consequence of a return to the gold standard. It is yet too early to know where the exchange value of gold will stabilize under post-war conditions, but, assuredly, the changes which have taken place since 1913 in the credit machinery erected upon gold and in the whole structure of the world markets have been so great that herein are found powerful additional arguments for avoiding any pre-war base for this index number, designed as it is for a highly specialized purpose. It is hoped that the shift of the base to 1923–25 will



prevent the confusion which might otherwise arise in comparing this with other published index numbers.

The new index is presented on the accompanying chart. For the purposes of testing it, the index of the United States Bureau of Labor Statistics has been converted to the 1923–25 base, and is shown with it. The sharpness with which the price swings are outlined by the new measure stands out clearly. The timing of the changes favors it rather than the more general index of the Bureau. This advantage appears in most cases. In addition is the fact that as the prices are taken as of the middle of the month and the computation is so short in form, the results may be made available about thirty days prior to the publication of the Bureau's Index.

A price decline begun in February, 1920, and well under way by the middle of May is a picture quite different from the continuous rise throughout the first five months of that year which is shown by the Bureau of Labor Statistics or the Dun Index. At the same time the definite improvement in business sentiment which took place in the summer of 1921 is clearly marked. The strong upward trend from this time to the end of 1925 is corroborated to some extent by both Dun's and Bradstreet's compilations. At the same time this new measure has not failed to define sharply the spirit of business optimism and subsequent reaction which characterized the spring months of 1923, 1924 and 1925, the hesitation in the spring of 1926, the subsequent

activity and the slowing up of business during the closing months of that year. Because it was felt that failure to measure the general level of prices might be the major weakness of an index so derived, the manner in which it fluctuates above and below the more general measure is very reassuring.

To check the results and throw some light on the reason for the differences between the two series Table III was worked up, which shows an index number of the price of each commodity made by comparing directly its average price in the three-year base period with the price recorded for the peak month, January, 1920, and with its average monthly price in the years 1919 and 1921.

TABLE III

RATIO OF PRICES FOR INDIVIDUAL COMMODITIES AT SPECIFIC PERIODS TO THE AVERAGE PRICE FOR THE SAME COMMODITY DURING THE THREE YEARS 1923–25

Commodity	January, 1920	Monthly average, 1919	Monthly average, 1921
Rosin	277	230	63
Hides	254	252	87
Silk	234	125	86
Sugar	226	126	82
Wheat	207	180	106
Silver	195	166	94
Burlap	181	154	55
Petroleum	175	133	98
Linseed oil	175	176	68
Wool	163	133	64
Hogs	158	194	91
Corn	157	166	61
Cotton	152	119	53
Cattle	142	157	83
Copper	138	136	91
Steel:			
Heavy melting scrap	135	97	71
Finished steel composite	121	120	97
Zinc	132	102	68
Coffee	128	123	51
Tin	126	130	60
Rubber	124	114	41
Bituminous coal	118	111	125
Lead	106	70	56
Arithmetic average of relatives	166	144	76
National Bank of Commerce index	168	141	75
U. S. Department of Labor index	151	134	95

The major discrepancy which occurs between this new price index and that of the Bureau of Labor Statistics is in the year 1921. It seems quite clear from the comparisons made in the table that the difference in this case arises from the character of the prices studied and not from the method of calculation. Certainly as far as these major commodities go the difference in prices between the 1923–25 average and the level of 1921 was much greater than 5 per cent in the majority of cases. Both the Bradstreet and Dun series point to a considerably wider difference.

As far as 1919 is concerned the relatives suggest that the higher levels indicated were due in considerable part also to the character of the markets from which prices were used. An additional explanation may be found in another feature which is one of the distinguishing characteristics of this index number.

During 1919 many of the major commodities were still under control. Sugar prices, for instance, were quoted unchanged throughout the year. Manifestly these fixed quotations were of no value in outlining current business tendencies and by the regular rule of calculation adopted for this index were discarded during this period. Naturally the result must be an index that moved higher than others which included these commodities while they were under control. For a considerable part of 1919 so many prices were fixed that the list became somewhat narrower than it might have been deemed wise to use under ordinary working conditions, but as the purpose of including that year at this time is mainly experimental the series was worked out without any change in either list or method.

The figures for January, 1920, are given for the interest there may be in checking the index at the indicated peak of prices. Again there is evidence for charging the difference between the series largely to the character of the prices included in each. And the difference of 11 per cent cannot be considered extraordinary in view of the sweeping movements of prices which centered around this point. Dun's Index, changed over to the same base, would range farther below the Bureau's Index than this does above. How easily such differences could occur is clearly manifested in the wide scatter of the individual prices themselves, which renders vague the meaning of any average founded upon them.

On the whole, therefore, the index appears to have justified itself to an extent which indicates merit for the method used in its construction.

A MEASURE OF STABILITY WITH A DISCUSSION OF CERTAIN OF ITS IMPLICATIONS

By Harry Sylvester Will

The subject of this disquisition will be discussed under three major heads: first, the measurement of stability; second, the predictive value of approximation equations; third, an analytical weight for use in the combination of observations.

I

The tendency of a statistical series to approximate a central value is ordinarily discussed from the point of view of variability. It is, however, sometimes of value to approach the subject from the standpoint of the stability of the series. We shall accordingly derive a formula for the measurement of stability.

Let $x_1, x_2, \ldots x_n$ be a series of observations with respect to which we make the following assumptions:

(a) That all are on similar objects and, therefore, in terms of the same unit.

(b) That each x_k, $k=1, 2, \ldots n$, is positive.

(c) That each x_k is the mean of a system of approximately normally distributed variates. That is, we assume that the errors in x_k are accidental in character.

(d) That the variable x_k is continuous.

These assumptions are manifestly in accord with the character of empirical observations.

Now we know that the stability of the series of variates x will be at a maximum when its dispersion is at a minimum; and we know that its dispersion will be at a minimum when the relative standard error of its mean, that is, the standard error of the mean of the relatives x/m, has a minimum value. We seek, then, the conditions which determine this minimum value.

We write accordingly

$$\Theta^2=\frac{\sigma^2}{nm^2}=\frac{\Sigma(x-m)^2}{n^2m^2}=\frac{\Sigma x^2-2m\cdot\Sigma x+nm^2}{n^2m^2}=\frac{\Sigma x^2}{(\Sigma x)^2}-\frac{1}{n}. \qquad (1)$$

Differentiating partially with respect to n, we obtain

$$\frac{\delta\Theta^2}{\delta n}=n^{-2}. \qquad (2)$$

With respect to x, we differentiate partially as follows: Let $u=\Sigma x^2$; and let $v=\Sigma x$. Then

$$\frac{\delta\Theta^2}{\delta x}=\frac{\delta}{\delta x}\left(\frac{u}{v^2}\right)-\frac{\delta}{\delta x}\left(\frac{1}{n}\right)=\frac{v^2D_xu-uD_xv^2}{v^4}=\frac{vD_xu-2uD_xv}{v^3}. \tag{3}$$

If we assign to each variable x_k the increment Δx_k, the function u will receive the increment Δu, and we have

$$\begin{aligned}u+\Delta u&=(x_1+\Delta x_1)^2+(x_2+\Delta x_2)^2+\ \ldots\ +(x_n+\Delta x_n)^2\\&=x_1^2+2x_1\Delta x_1+\Delta x_1^2+x_2^2+2x_2\Delta x_2+\Delta x_2^2\\&\qquad+\ \ldots\ +x_n^2+2x_n\Delta x_n+\Delta x_n^2.\end{aligned}$$

Ignoring Δx^2 as small in comparison with Δx, and subtracting $u=x_1^2+x_2^2+\ \ldots\ +x_n^2$, we have $u+\Delta u-u=\Delta u=2(x_1\Delta x_1+x_2\Delta x_2+\ \ldots\ +x_n\Delta x_n)$.

Since we have assumed that our observations are on similar objects and are stated in terms of the same unit, we may, *in the special case at hand*, therefore, do what is not generally allowable, that is, arbitrarily require that $\Delta x_1=\Delta x_2=\ \ldots\ =\Delta x_n=\Delta x$.

Writing $\Delta u=2(x_1+x_2+\ \ldots\ +x_n)\Delta x=2\Sigma x\Delta x$, we have, in the limit, $D_xu=2\Sigma x$. Similarly, we have $\Delta v=\Delta x_1+\Delta x_2+\ \ldots\ +\Delta x_n=n\Delta x$; and, in the limit, we get $D_xv=n$.

Substituting for u, v, D_xu, and D_xv in equation (3), we obtain

$$\frac{\delta\Theta^2}{\delta x}=\frac{2(\Sigma x)^2-2n\Sigma x^2}{(\Sigma x)^3}. \tag{4}$$

To obtain the conditions necessary for $\Theta^2=$ a minimum, we set equations (2) and (4) equal to zero, obtaining

$$n^{-2}=0;\ n=\infty. \tag{5}$$

$$\frac{2(\Sigma x)^2-2n\cdot\Sigma x^2}{(\Sigma x)^3}=0;\ \frac{(\Sigma x)^2}{\Sigma x^2}=n. \tag{6}$$

The interpretation of (5) and (6) is that the value of Θ^2 becomes a minimum, that is, 0, when $(\Sigma x)^2/\Sigma x^2=n=\infty$.

Obviously, the value 0 for the standard error of the mean of any statistical series implies that the series is single valued. For, setting $x=x_0$, we note that $\Sigma x_0=nx_0$ and that $\Sigma x_0^2=nx_0^2$; so that we have $(\Sigma x_0)^2/\Sigma x_0^2=n^2x_0^2/nx_0^2=n$, a constant.

We observe experimentally that the ratio $(\Sigma x)^2/\Sigma x^2$ diminishes in value in comparison with n as the variability of the given series increases. The difference $n-(\Sigma x)^2/\Sigma x^2$ can be made as small as we please but can-

not become less than 0. Therefore, $(\Sigma x)^2/\Sigma x^2 \doteq n$ as limit as the stability of the given series increases. That is,

$$n\beta^2 = (\Sigma x)^2/\Sigma x^2 \doteq \lim\ n. \tag{7}$$

Dividing by n and taking the root, we obtain

$$\beta = \Sigma x/\sqrt{n\Sigma x^2} = m/\sqrt{\Sigma x^2/n} \doteq \lim\ 1. \tag{8}$$

In equation (8), the Greek letter beta has been assigned as the symbol for what may be termed the *coefficient of stability* for a given series of variates. Defined, β may be said to measure the concentration of the observations about their most probable value; or in other words, it measures the approximation of the relatives x/m to the value $m/m = 1$. Experimentally, it approaches 1 as limit as the variates of the given series approach their most probable value, and it approaches 0 as limit as the stability of the series diminishes.

When β is computed from a frequency table, the following relations are important. Let x_0 denote the mid-class value of the lowest valued class of the frequency table; that is, let $x_0 = (x_{0u} + x_{01})/2$, or the mean of the upper and lower limits of the zero class of the table. Let i be the arbitrarily chosen class interval of the table; that is, $i = (x_{01} + x_{gu}) : (g+1)$, or the lower limit of the 0 or lowest class plus the upper limit of the gth or highest class divided by the number of classes to be formed. Then, letting F signify the class frequency, and Z the index value of classes 0 to g, we have

TABLE I

Discount Rates of Berlin (x_1) Discount Rates of Oslo (x_2)

1923	x_1	x_2	$100(x_1/m_1)$	$100(x_2/m_2)$	t_1	t_2	$(x_1-m_1)/\sigma_1$	$(x_2-m_2)/\sigma_2$
January	12	5	29	86	45	86	−0.85	−0.86
February	12	5	29	86	45	86	−0.85	−0.86
March	12	5	29	86	45	86	−0.85	−0.86
April	18	5	43	86	56	86	−0.68	−0.86
May	18	6	43	103	56	103	−0.68	+0.22
June	18	6	43	103	56	103	−0.68	+0.22
July	18	6	43	103	56	103	−0.68	+0.22
August	30	6	72	103	79	103	−0.33	+0.22
September	90	6	217	103	190	103	+1.40	+0.22
October	90	6	217	103	190	103	+1.40	+0.22
November	90	7	217	121	190	120	+1.40	+1.29
December	90	7	217	121	190	120	+1.40	+1.29

$m_1 = 41.5$ $\sigma_1 = 34.60$ $\beta_1 = 0.7685$ $\omega_1 = 0.0185$
$m_2 = 5.8$ $\sigma_2 = 0.93$ $\beta_2 = 0.9932$ $\omega_2 = 0.1702$

$$\Sigma x = i \cdot \Sigma FZ + nx_0;\ \Sigma x^2 = i(i \cdot \Sigma FZ^2 + 2x_0 \cdot \Sigma FZ) + nx_0^2. \tag{9}$$

From equations (9), the computer will note that β cannot be computed from the class indexes, class frequencies, and class interval alone. The

constant x_0 is indispensable because the coefficient of stability treats observations as *relative to* their mean value. This consideration does not hold in the computation of the standard deviation from a frequency distribution because, in this case, observations are treated as *deviations from* the mean.

It is interesting to note that the function β may be generalized in the form

$$\beta^b = (\Sigma x)^b : (n^{b-1}\Sigma x^b), \tag{10}$$

where the values of b are positive or negative integers or zero. The function $n\beta^2$ will presently be shown to be of special interest.

We find a simple application of the measure β in the construction of index numbers. There are two fundamental type ratios which are used in the making of indexes: first, the type $(x-m)/m$ and, second, the type $(x-m)/\sigma$. The one shows change as relative to the most probable magnitude of the observations, while the other shows change as relative to their general variability. The first is usually shown as positive relatives with m set equal to 100 and is fairly well understood by the laity. The second is usually shown in the form of positive and negative ratios and is not so easily understood by the layman.

We shall now show how both of these correction principles can be made use of in the construction of a single index. The procedure is simple. First, correct the series for the general magnitude of the observations by forming the ratios $(x-m)/m$. Second, correct for the stability of the series by multiplying these ratios by the coefficient β. The two operations may be combined into one by noting that $\beta/m = \Sigma x/\sqrt{n\Sigma x^2} \cdot (n/\Sigma x) = \sqrt{n/\Sigma x^2} = \omega$, let us say. To reduce the series to the form of positive relatives with $m = 100$, we write the equation of transformation as

$$t = 100 + (x-m)100\omega. \tag{11}$$

Transformation by the factor omega seems appropriately described as a process of modulation.

Table I shows a comparison of two series by means of three transformations: (a) simple relatives; (b) modulated indexes; (c) sigma ratios. The first transformation yields a comparison on the assumption of equal means, the third, on the assumption of equal stability, and the second, on the assumption of equal means and equal stability.

In modulating a series, the parameters m, β, and ω may be determined either from a fixed base period or from a moving period. The first would give results reflecting the influence of secular trend, while

the second would yield results neglecting such influence. In either case, the length of period selected should be such as to give definite significance to the constants used.

II

We shall now proceed to develop a measure which will be of service in determining the precision with which an approximation equation can be used for purposes of estimate. By an approximation equation is meant the mathematical representation of the relationship existing between the series under investigation as determined by the criteria of least squares.

Let us begin with stating the functional relations between β and ω on the one hand, and m and σ on the other.

$$v^2=\sigma^2/m^2=(\Sigma x^2/n-m^2)/m^2=n\Sigma x^2/(\Sigma x)^2-1=1/\beta^2-1. \tag{12}$$

Therefore,

$$\beta^2=1:(v^2+1)=1:[(\sigma/m)^2+1]. \tag{13}$$

Also, relating ω reciprocally to m and σ, we have

$$1/\omega^2=\Sigma x^2/n=\Sigma x^2/n-m^2+m^2=\sigma^2+m^2. \tag{14}$$

Denoting the ordinates of the fitted curve by z, let us substitute $(p'')^2$ for β^2, and $s^2=\Sigma(x-z)^2/n$ for σ in equation (13), obtaining

$$(p'')^2=1:[s^2/m^2+1]. \tag{15}$$

We now have a relation involving the precision with which a curve fits the empirical ordinates, the arithmetic mean of these ordinates, and their scatter about the curve. In this equation s may be regarded as being measured in terms of the unit m. Since s ordinarily takes small values in comparison with m, we should, in practice, find p'' to be a crude instrument for measuring the precision of approximation. For this reason, we shall employ as our unit $m/\sqrt{1000}$.

Writing $u'=\sqrt{1000}\cdot s/m$, we obtain the formula

$$p'=1:\sqrt{(u')^2+1}. \tag{16}$$

Noting that $(u')^2=1000s^2/m^2$, we see the practical convenience of using $m/\sqrt{1000}$ as our unit in the measurement of scatter rather than some other small fraction of m. A few values of p' and p'' corresponding to given values of s/m are given in Table II to show the advantage of p' over p'' as a measuring instrument.

TABLE II

$s:m$	p'	p''
0.001	0.9995	0.9999+
0.010	0.9533	0.9999+
0.050	0.5345	0.9990
0.100	0.3015	0.9950
1.000	0.0316	0.7072

Let us now compare the properties of p' with those of r, the coefficient of correlation. If we write the formula for the latter as

$$r^2 = 1 - s^2/\sigma^2, \tag{17}$$

by simple operations we may easily deduce from equations (16) and (17) the fundamental relation between r and p' as

$$r^2 = 1 - (1/(p')^2 - 1) : 10^3 v^2. \tag{18}$$

In this equation, since v as well as p' and r is variable, we see that p' cannot be predicted from r nor r from p'.

Both r and p' measure the approach of the relationship between variates to the form of a mathematical function of a specified type. The signification of the two measures, however, is not the same. For if we write the equation of regression of y on x in the form

$$y = a + bx + cx^2 + \ldots + lx^n, \tag{19}$$

we see that when $s = \sigma$, the coefficients of the first and higher powers of x vanish, with the result that $r = 0$ when the value of σ is finite and becomes indeterminate when $\sigma = 0$. Under the condition $s = \sigma$, p', on the other hand, always takes definite and significant values. From this it is clear that the common assumption that r is a sufficient measure of the value of an approximation equation for purposes of estimate is in error.

The distinction between r and p' as devices for the interpretation of relationship becomes still more evident when we note that in the one case the value of the measure derives from the ratio of the error of estimate to the variability of the observations, while in the other it derives from the ratio of the error of estimate to their most probable value.

To illustrate, suppose we are interested in comparing the prices of two commodities, y_1 and y_2, with their supplies, x_1 and x_2. Let the regression equations be of the form $y = f(x)$; and let us suppose that $s_1 = s_2$. Evidently, when $\sigma_1 = \sigma_2$ and $m_1 \neq m_2$, r_1 and r_2 fail to reveal the significant difference in the precision of estimates from the regression

TABLE III
EXAMPLES SHOWING COMPARATIVE VALUES OF r AND p

Ex.		Variate	m	s	σ	r	p	p'
1	x	Weekly earnings of agricultural laborers	191.3d	15.4d	20.5d	0.66	0.368	0.366
	y	Per cent of population receiving poor relief	3.67	0.97	1.29	0.66	0.126	0.119
2	x	Per cent male paupers over 65 years	16.73	6.07	6.45	0.34	0.092	0.087
	y	Ratio of outdoor to indoor relief	3.86	2.80	2.98	0.34	0.047	0.044
3	x	Age of husband	30	2.078	4.02	0.856	0.419	0.417
	y	Age of wife	26	2.17	4.17	0.856	0.358	0.355
4	x	Load factor	21.4	8.038	9.39	0.517	0.091	0.084
	y	Income per Kw. H.	3.45	1.68	1.95	0.517	0.074	0.066
5	x	First throws of dice	5.95	1.545	1.73	0.45	0.126	0.121
	y	Second throws of dice	6.11	1.53	1.72	0.45	0.131	0.125
6	x	Length of carapace of shrimp	250	3.754	6.73	0.83	0.903+	0.903−
	y	Length of post-spinous portion	178	2.90	5.18	0.83	0.889+	0.889−
7	x	Duration of eclipse in hours	10.4	2.5	5.26	0.881	0.146	0.130
	y	Orbital period in hours	82.7	26.4	55.6	0.881	0.119	0.099
8	x	Debits	29.13	12.277	22.19	0.833	0.094	0.075
	y	Clearings	39.5	13.17	23.95	0.833	0.111	0.093
9	x	Taxable income in thousands	83.8	29.72	83.8	0.935	0.125	0.089
	y	Taxable motor vehicles in thousands	177	60.6	171.4	0.935	0.128	0.092
10	x	Height of midparent	68.31	1.336	1.45	0.389	0.850+	0.850
	y	Height of offspring	68.22	2.034	2.208	0.389	0.728	0.725
11	x	Votes of men in per cents	60.8	7.304	14.71	0.868	0.261	0.255
	y	Votes of women in per cents	60.6	8.68	17.35	0.868	0.224	0.216
12	x	Irrigation water in inches						
	y	Tons per acre	7.48	1.36	2.27	0.80	0.179	0.171
13	x	Depth below surface						
	y	Velocity of Mississippi River, feet per second	3.1762	0.0023	0.0917	0.9996	1.000−	0.999+
14	x	Seven series						
	y	Trend of cotton prices	148.3	5.305	27.344	0.981	0.670	0.662

equations y_1 and y_2 which are implied in the necessary difference between the values of p_1' and p_2'. The case is reversed when $\sigma_1 \neq \sigma_2$ and $m_1 = m_2$; for then a significant difference between the values of r_1 and r_2 appears, while p_1' is seen to equal p_2'.

The foregoing analysis suggests that the usefulness of p', as a measure of the validity of the assumption that the empirical ordinates lie on the fitted curve, would be improved if it were corrected for the stability of the series from which it is determined. This we can easily do by making use of the function ω developed in Part I. Writing $u = \omega s\sqrt{1000}$, we have

$$p = 1 : \sqrt{u^2+1}. \tag{20}$$

The coefficient p we may define as *the precision with which a function of the form* $y = f(x_1, x_2, \ldots x_n)$ *may, when fitted by the method of least squares, be employed to represent a series of observations.* It is to be distinguished from the modulus of precision $h = 1/\sigma\sqrt{2}$, which is a parameter of the normal curve of error. By inspection we see that as $s \doteq 0$, $p \doteq 1$; and that as $s \doteq \infty$, $p \doteq 0$.

Table III has been prepared to show the values of p, p', and r for given values of s, m, and σ, taken for the most part from examples of

correlation in standard works on mathematics and statistics. We note (a) that high values of p are not necessarily associated with high values of r, and conversely; also, (b) that the values of p are slightly higher than the values of p'. The apparent exceptions to (b) are due merely to the rounding of decimals, for $p = p'$ only in case $p = 1$. Occasionally, for highly unstable series, we may expect to find p considerably greater than p'.

We take pains to point out again that, since the coefficient of correlation and the coefficient of precision are not the same in their signification, the one cannot be employed as a substitute for the other. The first is essentially a measure of the extent to which change in one variable is dependent on change in the other; while the second must be regarded as a measure of the extent to which the values of one variable are predictable from the values of the other. Where r is given, it would usually seem desirable to state the values of p also. Certainly, the common practice of giving r without mentioning the value of s is altogether to be discouraged.

III

Where r distinct samples, each consisting of n_r observations on a given attribute, are drawn from a total population of N objects, it is necessary to give definite weights to the results of each sample before they can be combined to give the most plausible estimate for the whole population. If we take the arithmetic mean as giving the most probable value for a single sample, the combination of results is effected according to the following equation:

$$\begin{aligned}(c_1m_1+c_2m_2+ \ . \ . \ . \ +c_rm_r) &: (c_1+c_2+ \ . \ . \ . \ +c_r) \\ &= m_1 \cdot c_1/\Sigma c+m_2 \cdot c_2/\Sigma c+ \ . \ . \ . \ +m_r \, . \, c_r/\Sigma c=A, \quad (21)\end{aligned}$$

where the coefficients c_k, $k=1, 2, \ . \ . \ . \ r$, are termed the absolute weights of the observations m_k and the coefficients $c_k/\Sigma c$ are termed their relative weights.

These weights may be regarded as measuring the extent of our confidence in the validity of the quantities m_k. The ways in which the values of c are usually determined are here stated: (a) The extent of observation, the number involved, is assigned as weight; $c=n$. This gives what is commonly called the weighted average. (b) The weight is computed by taking the product of the number of observations involved and the precision with which the observations are made. If q is assigned as the symbol for this weight, we have $c=q=n/\sigma^2$, where the reciprocal of the squared standard deviation is used as the measure

of precision of observation. The derivation of the weight q from the principles of least squares is well known.

TABLE IV

DATA FROM WHICH PARAMETERS WERE COMPUTED

Class	F_1	F_2	F_3	ΣF
1– 5	9	6	2	17
6–10	1	3	3	7
11–15	6	9	3	18
16–20	5	6	3	14
21–25	9	5	3	17
26–30	6	5	2	13
31–35	8	6	4	18
36–40	13	6	2	21
41–45	7	3	3	13
46–50	9	4	6	19
51–55	9	7	3	19
56–60	12	6	3	21
61–65	9	9	2	20
66–70	9	2	0	11
71–75	9	2	2	13
76–80	12	6	5	23
81–85	5	3	3	11
86–90	10	9	3	22
91–95	9	3	3	15
96–100	17	6	7	30
n	174	106	62	342
m	56.213	49.226	54.065	
σ	28.187	29.278	29.844	
β	0.89392	0.85947	0.87547	
q	0.21901	0.12366	0.06961	
w	139.04	78.30	47.52	

We shall now show that the weighting principles stated in (a) and (b) can be adequately expressed in terms of a single fundamental formula. Let it be supposed that two samples are drawn from a population consisting of an infinite number of individuals, each of which possesses the attribute L. The first sample consists of a single occurrence of the magnitude L_1, while the second sample shows n occurrences of the magnitude L_2. From the first sample, we have no basis for supposing the existence of any magnitude other than L_1, and from the second, no basis for supposing the existence of any magnitude other than L_2. The strength of our belief in the comparative validity of the magnitudes L_1 and L_2 as evidence of the true constitution of the total population is fairly represented by the numbers 1 and n, which may be taken as the weights of the two observations.

Since no sample figuring in the final estimate could show less than a single observation, the number 1 would seem to be the natural minimum value which should be assigned as the weight of any such observation; and likewise, since no sample could show more than n observations, the number n would seem to be the natural maximum value which should be assigned as weight. We should expect this maximum limit to be at-

tained only when the observations are made with absolute precision, that is, when all the n observations are of the same magnitude, as L_k.

If all the observations of a given sample are not of the same magnitude, less than absolute precision of observation is indicated, and it becomes necessary to correct the weights for this lessened precision. This may be done by introducing the factor $1/\sigma^2$. The use of this measure, however, leads to certain theoretical inconsistencies. For if a first sample shows n_1 occurrences of the magnitude L_1, and a second sample shows n_2 occurrences of the magnitude L_2, σ_1 and σ_2 both vanish and the absolute weights q_1 and q_2 both become infinite, while the corresponding relative weights $q_1/\Sigma q$ and $q_2/\Sigma q$ take the value 1.

TABLE V

AVERAGES OBTAINED FROM WEIGHTS COMPUTED UNDER VARIOUS ASSUMPTIONS

Subscript	A_n	A_q	A_w
1+2	53.568	53.691	53.696
1+3	55.648	55.694	55.666
2+3	51.012	50.968	51.054
1+2+3	53.657	53.754	53.762

If we require that $n_1 \neq n_2$, we see that these results are clearly contradictory; for we could accord an infinite absolute weight only in case the sample itself were infinite and consisted of identical results; and likewise, the relative weight 1 could be assigned only in the case when but a single valid sample was at hand. The point to be noted here, of course, is that the weights q yield identical results when they should show significant differences.

In texts on the subject, the use of the weight q is usually qualified by the condition that the number of observations must be large enough to guard against a run of luck. Strictly, of course, there is no number large enough to prevent a run of luck that will give unduly large value to the weight q, though we admit that increasing the number of observations does diminish the chance that an unrepresentative sample will be drawn.

By employing β^2 instead of $1/\sigma^2$ as the measure of precision, we are able to deduce a weight which will overcome the theoretical difficulty just set forth. Writing $c=w$, we have

$$w = n\beta^2 = (\Sigma x)^2/\Sigma x^2. \tag{22}$$

The values of w, as we have seen in Part I, cannot lie outside the limits 1 and n.

The use of the weight w instead of q in such a case as that cited in the preceding paragraph would obviate the inconsistency there developed. For, instead of the infinite values of q_1 and q_2, w_1 and w_2 would attain the finite limits n_1 and n_2 and the corresponding relative weights would assume unlike values. It thus appears that the principle of weighting stated in (a) is but a special case of the principle stated in (b), if $w = n\beta^2$ is employed instead of $q = n/\sigma^2$ as the index of weight. Principle (a) in reality assumes that the mean observation m is made with absolute precision.

The index w, it will be noted, is always a pure number. In this respect, it is unlike q, the value of which varies with the magnitude of the unit of measurement employed in making the observations.

To illustrate some of the values which may be taken by the weights n, q, and w, the following data were employed. Three observers were asked to draw samples from a case of thirty-five sections containing, in all, twenty-five thousand cards. All that was known of the constitution of the total population was that each card registered the score of a student on the university intelligence test and that the total distribution of scores was roughly normal. Each sample was drawn in a random manner and all cards were kept in their original order in the file. The exact manner in which each sample was drawn was not ascertained. Hence, the *a priori* assumption that the total record constitutes but a single homogeneous sample is excluded. Table IV supplies a record of observations for each sample together with the constants computed from each distribution. Computations were made from the original distributions and not from the frequency table, however. Table V gives the adjusted means (A) as computed from the weights recorded. The figures in the column headed "subscript" indicate the samples from which the constants employed in computing the adjusted means were determined.

The results that are shown in Table V are in no wise striking; nor, from the nature of the samples employed, was it to be expected they would be. As here, the results obtained by the weight w will usually agree more closely with those obtained by the weight q than they will with those obtained by the weight n. The exceptions to this rule will occur when a sample showing a fortunate clustering about the mean in the values of observations will unduly influence the magnitude of q.

FACTORS AFFECTING CHANGES IN SHORT TERM INTEREST RATES[1]

By W. Randolph Burgess

In the study of causes of changes in short term interest rates the usual procedure has been to study the correlation of interest rates with various factors of the business cycle such as car loadings, production, bank debits, security prices, etc. Such a study involves us in the computation of secular trends and seasonal variations, in price corrections, and in problems of lags and leads, and the results, while they appear plausible for brief periods are usually most discouraging when viewed in broader perspective.

It is the purpose of this paper to suggest and illustrate another approach to the study of causes of interest rate changes. It is not a wholly new approach. President Kinley of the University of Illinois used some such approach in his volume on the *Independent Treasury* in the National Monetary Commission report, and a number of today's students of the money market are using the approach somewhat. But, in general, this approach is not employed by most students of the money market and interest rates. It is not understood by financial writers or generally by bankers. The approach is simple: no coefficients of correlation are required; the only mathematical operations involved are addition and subtraction, with now and then a bit of multiplication.

The proposal is that the discovery of causes of changes in short term interest rates is mainly a problem of simple accounting—the subject matter of which is bank reserves.

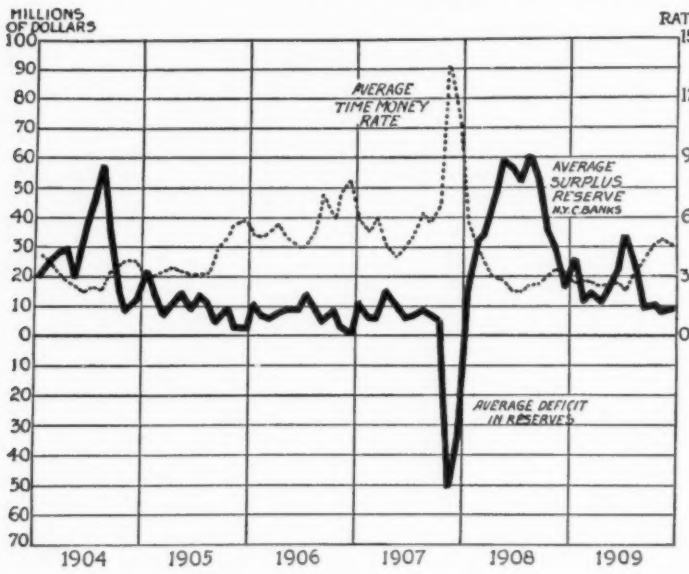
The theory of this approach may be stated briefly and in truisms. Banks are the custodians of money in this country. When the bankers have much money to lend, money rates tend to be easy; when they have little to lend, money rates tend to be firm. The amount a banker can lend depends upon his reserve position. Therefore, the reserve position of the banks of the country determines short term money rates, and the causes of changes in money rates are to be found in the causes of changes in the reserve position of banks.

The theory will be clearer if illustrated. Chart I shows the relationship between the average monthly time money rate and the average surplus or deficit in reserves of the New York City Clearing House

[1] Presented at the eighty-eighth annual meeting of the American Statistical Association, St. Louis, December, 1926.

CHART I

CLOSE RELATION OF THE TIME MONEY RATE TO THE SURPLUS OR DEFICIT OF RESERVES OF NEW YORK CITY BANKS BEFORE THE RESERVE SYSTEM



banks for several years prior to the establishment of the Federal Reserve System. It will be observed that there is a close inverse relationship between the two lines shown. When there were large surplus reserves, money rates were low; and when reserves were low, money rates were high.

One impressive feature of the diagram is the narrowness of margin of reserves. For many months average surplus reserves were less than $10,000,000. In November, 1907, it was an average deficit of only $50,000,000 which signalized the money panic of that autumn. The extraordinary feature of this narrowness of margin in reserves is that it represents the margin not simply for New York City but in considerable measure for the country as a whole, for in those days New York was the principal central reserve city for the country. The reserves of thousands of out-of-town banks were largely deposited with the New York City banks. Thus a $10,000,000 surplus represented the surplus for a large part of the country. With a total of money in circulation or in banks of nearly $3,000,000,000 at that time, a difference of $10,000,000, or one-third of one per cent, made the difference between money ease and money stringency. Here is an interesting illustration of the law of marginal differences. A change of one-third

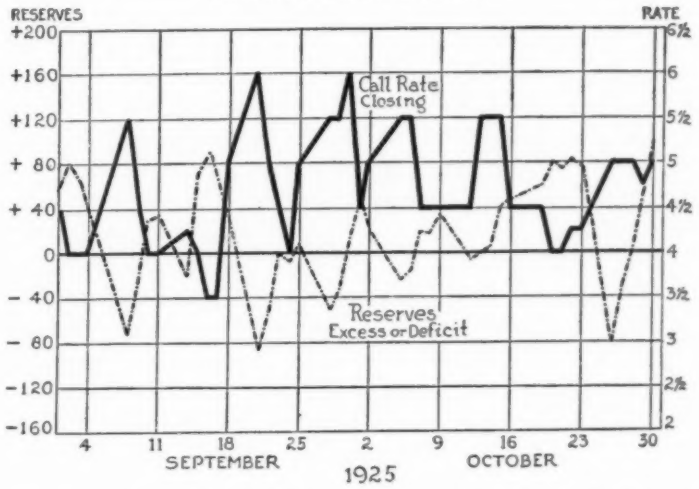
of one per cent in the supply of or demand for funds may change the price 20 or 25 per cent.

But the important feature of the diagram is not the amounts of change in the two lines but the closeness of correspondence in their movements. There is nothing surprising about this closeness of correspondence but it is entirely logical in view of the circumstances. Time money is lent by banks. When they have large reserves, they are putting money into the market, and when they have inadequate reserves, they are withdrawing money from the market. There is thus a natural and direct relation between reserves and money rates.

Under the Federal Reserve System the situation is a bit more complicated. Generally speaking, there is no longer any such thing as a surplus in average bank reserves. Since reserves of the member banks have been pooled in the Reserve Banks, and these reserves may be used in emergencies and busy seasons by borrowings at the Reserve Banks, bankers no longer plan to carry surplus reserves themselves but plan to end each reserve period with their average reserves as close as possible to reserve requirements. In other words, they plan to keep all their money at work all the time.

CHART II

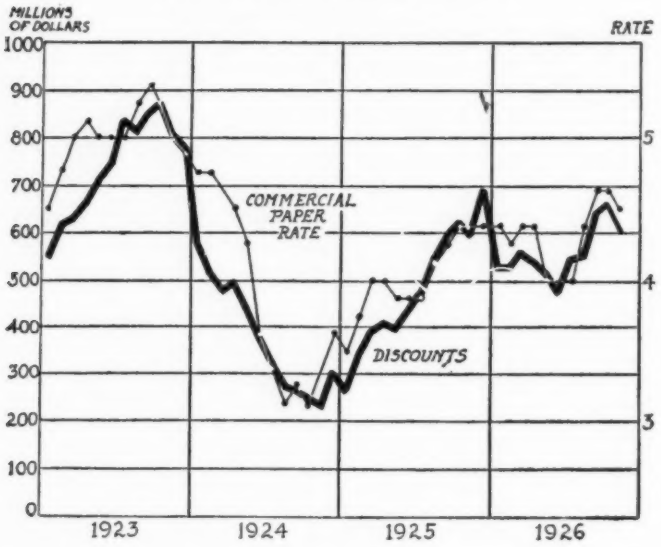
DAILY EXCESS OR DEFICIT IN RESERVES OF 23 NEW YORK CITY BANKS AND THE CLOSING CALL LOAN RATE



Reserves are only balanced, however, once a week in the large cities and twice a month elsewhere, and between these days of reckoning we may still observe the old interactions between money rates and reserves. This is illustrated in Chart II. which shows the daily accu-

CHART III

OPEN MARKET INTEREST RATE FOR PRIME 4–6 MONTHS COMMERCIAL PAPER AND AVERAGE DAILY BILLS DISCOUNTED FOR MEMBER BANKS BY ALL FEDERAL RESERVE BANKS



mulated excess or deficit of reserves of the 23 largest New York City banks, and the call loan closing rate. The inverse correlation between reserves and money rates is similar to that of Chart I. The explanation is the same, except that the banker today has perhaps a little more definite goal in his endeavor to bring out his average reserves at the end of the week as close as possible to his legally required reserves. Each week is now a complete picture in itself and we must consider other factors than the reserve position alone in any study of week to week or month to month rate changes.

The other element in the picture is the borrowing of member banks at the Federal Reserve Banks. The relation between such borrowing and monthly money rates is shown in Chart III. Here again we have a remarkably close relationship.

We may best fit this borrowing at the Reserve Banks into our picture of bank reserves and money rates, if we think of this borrowing as an employment of bank reserves. When a banker comes to the Reserve Bank and borrows, giving us his secured note or his customers' paper, he is putting into use reserve funds. It is something as though in Chart I reserves had been paid out until the figures showed a deficit in reserves. The principal difference is that the Federal Reserve Act

regularized and made safe the putting to use of legal reserves to a limited extent and under careful restrictions.

At any rate, the figures and our own experience make it clear that the bankers treat borrowing from the Reserve Banks as a temporary expedient. The effect on money rates is somewhat similar, although less acute, to the effect of using up surplus reserves in the old days. When they are borrowing from the Reserve Banks the member banks tend to lend less freely, to withdraw funds from the market rather than put funds into it. The consequence is the close parallel between money rates and borrowing, which Chart III shows.

From these charts and from the related facts it appears that there is a direct relationship between money rates and the reserve position of the banks, including under the term "reserve position" the amount of borrowing from the Reserve Banks. It is primarily the position of his reserves which leads the banker to take the action which brings about higher or lower money rates. There are, of course, some other factors in the banker's decision, the most important other factor being his judgment as to the future. For example, in December banks borrow very heavily from the Reserve Banks to meet Christmas and year-end demands for funds, but the commercial paper rate is not affected because bankers know that the demand is temporary. In the long run this factor of anticipation is not a very important one, for the banker is not a speculator and his decision is generally based on his current reserve position.

We may put the general thesis another way by saying the bank reserve position is the focal point at which monetary supply and demand meet. The reserve position gives us a quantitative measure of marginal differences.

A reasonable conclusion from this discussion would seem to be that this is the point at which a study of money conditions and money theory may concentrate with every hope of profitable results. Fortunately, we now have available excellent figures which reveal the changes in the reserve position of the banks of the country and the reasons for those changes.

In the old days we had in the weekly report of the New York City clearing house banks excellent figures for the reserve position of those banks. But they did not reveal the reasons for changes. There were no current bank reports for the rest of the country. Gold movements and changes in currency in circulation were reported only once a month.

Today we have weekly reports of banking changes in all districts, weekly gold reports, and, in the Federal Reserve statement, a weekly summary of practically every influence on bank reserves. With the

use of these statements it is possible to carry forward a current accounting of the state of the money market, which will have a high degree of accuracy. We are currently making such an accounting at the Federal Reserve Bank of New York, and have been for over two years.

The method is illustrated by Table I, which attempts a quantitative explanation of the interest rate changes since the autumn of 1924, when the current movement of rates began. The principal influences on bank reserves since that time are shown in two periods. A rather arbitrary division of influences is made between supply and demand.

TABLE I
CHANGES IN THE DEMAND FOR AND SUPPLY OF RESERVE FUNDS
(In millions of dollars)
August 30, 1924–October 31, 1925

	New York	Other	Total
Demand:			
Currency	+ 31	+ 92	+123
Reserves	+ 38	+119	+157
	+ 69	+211	+280
Supply:			
Gold	−123	.....	−123
Transfers	+197	−197	.....
F. R. credit	− 5	+424	+419
	+ 69	+227	+296

October 31, 1925–October 30, 1926

	New York	Other	Total
Demand:			
Currency	+ 8	+24	+32
Reserves	− 5	+15	+10
	+ 3	+39	+42
Supply:			
Gold	+45	....	+45
Transfers	−69	+69	.....
F. R. credit	+33	−13	+20
	+ 9	+56	+65

The principal demand factors have been increases in demand for currency and credit. Increases in credit are reflected in the reserve position in the increases in the amount of reserves required to be maintained. This increase in reserve requirements can, of course, be analyzed further by the use of member bank reports, which show how much of the additional reserves required are due to increases in demand deposits, which require on the average 10 per cent reserves, and how much to increases in time deposits, which require 3 per cent reserves. And we may also trace the changes in loans and investments which parallel these changes in deposits.

It may be well to observe the important place that currency occupies in the demand side of the reserve equation. When we ordinarily discuss changes in interest rates we use the terms demand for and supply of funds usually thinking, I believe, of bank credit. This is naturally so because the amounts of credit used in business are so much larger than the amounts of currency. When we are thinking of bank reserves, however, the true importance of currency appears. The withdrawal of $100 of currency requires $100 of bank reserves. Whereas $100 of demand deposits require only about $10 of reserves. Thus while changes in currency are usually smaller than changes in deposits, each currency change has ten times the effect on reserves as a like change in deposits. You seldom find in discussions of the money market any discussion of currency changes, and yet such changes are a major influence upon money conditions.

On the supply side of the reserve equation the principal factors are gold and Federal Reserve credit. Increases in these items increase bank reserves and vice versa. It should, however, be noted that increases in supply through gold involve no pressure on the money market, whereas increases in Federal Reserve credit, if they take the form of member bank borrowing, do place some pressure on the market and tend to raise interest rates. Increases in Federal Reserve credit may also take the form of increases in holdings of acceptances and government securities in which case little or no pressure is placed on the market.

The changes in the supply half of the table do not precisely balance the changes in the demand half. This is because there are a number of other minor factors which have been neglected for the sake of simplicity. These other factors include changes in National bank note circulation, changes in government and foreign deposits in the Reserve Banks, changes in float carried by the Reserve Banks, etc., all of which should be included in any precise estimate but all of which may be omitted without seriously affecting the balance sheet.

The table may be interpreted briefly as showing a considerable increase in the demand for funds between September, 1924, and November, 1925, a decrease in the supply of gold and a consequent calling into use of additional amounts of Federal Reserve credit. The result has been a rise in interest rates.

By the use of the Federal Reserve and Member bank statements and statements for gold movements it is now possible to draw up from week to week and from month to month balance sheets of the influences in bank reserves, like Table I. It is our belief that this type of analysis reveals the direct causes of changes in short term interest rates.

ANALYSIS OF A U-SHAPED FREQUENCY DISTRIBUTION[1]

By Paul R. Rider, *Washington University*

The U-shaped frequency curve, although comparatively rare, is sometimes found in statistics. Pearson[2] states: "the 'prevalence of extremes' [which, of course, gives a U-shaped curve] may appear not only in meteorological phenomena but in competitive examinations, where the mediocre have occasionally sufficient wisdom to refrain from entering." The writer has noticed in certain mathematics examinations a pronounced tendency toward a gap in the grades obtained, a relatively large number of students making high or low grades with fewer or none acquiring grades intermediate in the scale. Pearson also calls attention to Galton's curve of "consumptivity,"[3] which is U-shaped. Yule[4] adduces, in addition to some of the examples already mentioned, a table showing the percentages of deaf-mutes among children of parents one at least of whom was a deaf-mute. The distribution is roughly U-shaped. Perhaps the best example of a U-shaped distribution is a certain set of data showing the degrees of cloudiness at Breslau,[5] that city whose birth and death records furnished Edmund Halley with the material for the construction of the first mortality table known.

To graduate a U-shaped distribution Pearson employs his generalized frequency curve of type I, whose equation is

$$y = y_0\left(1+\frac{x}{a_1}\right)^{\nu a_1}\left(1-\frac{x}{a_2}\right)^{\nu a_2}.$$

He has graduated the above-mentioned cloudiness data,[6] and the agreement between theoretical and observed frequencies is fairly good.

It would seemingly be impossible, or at best extremely unsatisfactory, to fit a Charlier curve to a U-shaped distribution, but the shape of the frequency curve suggests that it might be possible to fit each end with a Charlier curve of type B. This the writer has done, and the results obtained are somewhat better than those of Pearson.

[1] Presented to the American Mathematical Society, December 29, 1925.

[2] Karl Pearson, "Contributions to the Mathematical Theory of Evolution," *Philosophical Transactions of the Royal Society of London*, Vol. 186A (1895), pp. 364 ff.

[3] See Francis Galton, *Natural Inheritance* (1889), p. 174.

[4] G. Udny Yule, *An Introduction to the Theory of Statistics* (1922), p. 104.

[5] See Yule, *loc. cit.*, pp. 103 ff. See also Hugo Meyer, *Anleitung zur Bearbeitung meteorologischer Beobachtungen für die Klimatologie* (1891), p. 108. (The latter reference is taken from Pearson.)

[6] Karl Pearson, "Cloudiness: Note on a Novel Case of Frequency," *Proceedings of the Royal Society of London*, Vol. 62 (1897–98), pp. 287–290.

The cloudiness data are as follows:

ESTIMATED DEGREES OF CLOUDINESS AT BRESLAU DURING THE DECADE 1876–85

Cloudiness	0	1	2	3	4	5	6	7	8	9	10
Frequency	751	179	107	69	46	9	21	71	194	117	2,089

It is seen that there are eleven grades of intensity of cloudiness, ranging from 0 to 10, and indicating the part of the sky per decem covered by clouds. The frequencies are the respective numbers of days on which the given degree of cloudiness occurred.

In order to analyze this distribution let us separate it into two parts. It might seem desirable to dissect the distribution into two Poisson exponential functions after the manner in which Pearson [1] dissects a distribution which by hypothesis is composed of two normal distributions. This process, however, is exceedingly complicated, and in attempting to decompose a series into two Poisson exponentials by an analogous procedure one encounters certain mathematical difficulties which appear to be insuperable.

Moreover, it is easily ascertained that if either end of the cloudiness distribution is to be fitted with a Poisson exponential, an expansion to more than term must be employed.

However, even if it were theoretically possible so to resolve a U-shaped distribution into two constituent parts, practical considerations would lead us to reject any such process unless it were indeed extremely simple. For if the distribution may be separated, obviously only the tails of the two components overlap, and as the principal parts of the two components are quite distinct it is unnecessary to enter into any refinements in effecting the division. Let us therefore divide the cloudiness data somewhat arbitrarily into the following parts: [2]

PART I

Cloudiness	0	1	2	3	4	5	6
Frequency	751	179	107	69	45	4	1

PART II

Cloudiness	4	5	6	7	8	9	10
Frequency	1	5	20	71	194	117	2,089

[1] Karl Pearson, "Contributions to the Mathematical Theory of Evolution," *Philosophical Transactions of the Royal Society of London*, Vol. 185A (1894), pp. 71–110.

[2] For an example of the arbitrary breaking up of a frequency distribution, see Arne Fisher, *The Mathematical Theory of Probabilities* (1922), p. 244. Fisher fits each arbitrary part with a logarithmically transformed Charlier curve of type A. In the case of a U-shaped distribution there would seem to be much greater justification for an arbitrary division.

It will be recalled that for Charlier curves of type B [1] the generating function is Poisson's exponential

$$\Psi(x) = e^{-m} m^x / x!,$$

in which m is the mean of the distribution under consideration. The equation of the frequency curve is obtained by expanding in terms of differences of Ψ and is of the form

$$y = N[\Psi(x) + \gamma_2 \Delta^2 \Psi(x) + \gamma_3 \Delta^3 \Psi(x) + \ . \ . \ . \ ].$$

The constants N, γ_2, γ_3, . . . may be determined by the method of moments. It is readily found that N is the total frequency of the distribution, and that

$$\gamma_2 = \tfrac{1}{2}(\sigma^2 - m),$$

where σ is the standard deviation, and m, as already stated, is the mean. It will be unnecessary for our purposes to find any of the coefficients beyond γ_2, as we shall carry the expansion only to two terms.

The computation of these quantities for Part I is shown in Table I.

TABLE I

Cloudiness x	Frequency f	xf	x^2f
0	751	0	0
1	179	179	179
2	107	214	428
3	69	207	621
4	45	180	720
5	4	20	100
6	1	6	36
Total	1,156	806	2,084

$$N = 1156, \ m = \frac{\Sigma xf}{N} = \frac{806}{1156} = 0.697$$

$$\gamma_2 = \tfrac{1}{2}(\sigma^2 - m) = \tfrac{1}{2}\left(\frac{\Sigma x^2 f}{N} - m^2 - m\right)$$

$$= \tfrac{1}{2}\left[\frac{2084}{1156} - (0.697)^2 - 0.697\right] = 0.3097.$$

Making use of values of $\Psi(x) = e^{-0.697}(0.697)^x / x!$ which may be found in tables,[2] we develop the results shown in Table II.

[1] See C. V. L. Charlier, *Vorlesungen über die Grundzüge der mathematischen Statistik*, pp. 80 ff. See also Fisher, *loc. cit.*, pp. 271 ff.; or H. C. Carver, "The Generalized Normal Curve—Charlier Theory" in *Handbook of Mathematical Statistics* (edited by H. L. Rietz), pp. 114–119.

[2] Karl Pearson, *Tables for Statisticians and Biometricians*, part I, 2d edition (1924), pp. 113 ff.

TABLE II

x	$\Psi(x)$	$\Delta\Psi$	$\Delta^2\Psi$	$N\Psi$	$N\gamma_2\Delta^2\Psi$	$N(\Psi+\gamma_2\Delta^2\Psi)$	Observed frequency
0	0.4982	+0.4982	+0.4982	575.92	+178.36	754.3	751
1	0.3471	−0.1511	−0.6493	401.25	−232.46	168.8	179
2	0.1210	−0.2261	−0.0750	139.88	− 26.85	113.0	107
3	0.0281	−0.0929	+0.1332	32.48	+ 47.69	80.2	69
4	0.0049	−0.0232	+0.0697	5.66	+ 24.95	30.6	45
5	0.0007	−0.0042	+0.0190	0.81	+ 6.80	7.6	4
6	0.0002	−0.0005	+0.0037	0.23	+ 1.32	1.6	1
7	0.0000	−0.0002	+0.0003	0.00	+ 0.11	0.1	0

In working with Part II it is convenient to set $\xi=10-x$. The computation of the quantities N, m, γ_2 for Part II is shown in Table III.

TABLE III

Cloudiness x	$\xi=10-x$	Frequency f	ξf	$\xi^2 f$
10	0	2,089	0	0
9	1	117	117	117
8	2	194	388	776
7	3	71	213	639
6	4	20	80	320
5	5	5	25	125
4	6	1	6	36
Total	..	2,497	829	2,013

$$N=2497,\ m=\frac{\Sigma\xi f}{N}=\frac{829}{2497}=0.332$$

$$\gamma_2=\tfrac{1}{2}(\sigma^2-m)=\tfrac{1}{2}\left(\frac{\Sigma\xi^2 f}{N}-m^2-m\right)$$

$$=\tfrac{1}{2}\left[\frac{2013}{2497}-(0.332)^2-0.332\right]=0.182.$$

We now find values of $\Psi(\xi)=e^{-0.332}(0.332)^{\xi}/\xi!$, and proceeding as before obtain the results exhibited in Table IV.

TABLE IV

x	ξ	$\Psi(\xi)$	$\Delta\Psi$	$\Delta^2\Psi$	$N\Psi$	$N\gamma_2\Delta^2\Psi$	$N(\Psi+\gamma_2\Delta^2\Psi)$	Observed frequency
10	0	0.7183	+0.7183	+0.7183	1,793.60	+326.43	2,120.0	2,089
9	1	0.2369	−0.4814	−1.1997	591.54	−545.21	46.3	117
8	2	0.0398	−0.1971	+0.2843	99.38	+129.20	228.6	194
7	3	0.0046	−0.0352	+0.1619	11.49	+ 73.58	85.1	71
6	4	0.0004	−0.0042	+0.0310	1.00	+ 14.09	15.1	20
5	5	0.0000	−0.0004	+0.0038	0.00	+ 1.73	1.7	5
4	6	0.0000	−0.0000	+0.0004	0.00	+ 0.18	0.2	1

In Table V the theoretical frequencies obtained by adding the ordinates of the two Charlier components are compared with the observed frequencies and with the theoretical frequencies obtained by Pearson.

TABLE V

Cloudiness x	Ordinate of 1st Charlier curve y_1	Ordinate of 2d Charlier curve y_2	Sum of two Charlier components y_1+y_2	Observed frequency	Area of Pearson curve
0	754.3	.....	754.3	751	803
1	168.8	.....	168.8	179	142
2	113.0	.....	113.0	107	72
3	80.2	.....	80.2	69	60
4	30.6	0.2	30.8	46	51
5	7.6	1.7	9.3	9	50
6	1.6	15.1	16.7	21	55
7	0.1	85.1	85.2	71	60
8	.....	228.6	228.6	194	85
9	.....	46.4	46.4	117	153
10	.....	2,119.7	2,119.7	2,089	2,122
Total	.....	.....	3,652.9	3,653	3,653

Professor Carver, who read this paper in manuscript, suggested carrying the graduation to a second approximation.[1] To do this we break up the original distribution into two parts which are proportional to the first approximate graduation. The method of doing this is shown in Table VI, in which columns (1) and (2) give the theoretical

TABLE VI

x	(1)	(2)	(3) (1)+[(1)+(2)]	(4) Observed frequency	(5) (3)×(4)	(6) (4)−(5)
4	30.6	0.2	0.9935+	46	46	0
5	7.6	1.7	0.8172	9	7	2
6	1.6	15.1	0.0958	21	2	19
7	0.1	85.1	0.0012	71	0	71

frequencies of Parts I and II respectively, for our first approximation. Column (3) exhibits, for each class in which the two parts of the distribution overlap, the proportion of the theoretical frequency which belongs to Part I. Columns (5) and (6) show how the observed frequencies are apportioned to Parts I and II respectively for our second graduation.

Proceeding as before, we find that for Part I, $N=1161$, $m=0.716$, $\gamma_2=0.388$; and for Part II, $N=2492$, $m=0.323$, $\gamma_2=0.165$. The

[1] Professor Carver states that a U-shaped distribution may also be graduated by taking the reciprocals of the frequencies and treating the result as a simple unimodal distribution; he adds that it may be desirable to use "weighted moments" in such cases.

computation for Part I is shown in Table VII and for Part II in Table VIII. Results are summarized in Table IX.

It appears that the fit of the sum of two Charlier components is somewhat better than that of the Pearson curve. One's attention,

TABLE VII

x	$\Psi(x)$	$\Delta\Psi$	$\Delta^2\Psi$	$N\Psi$	$N\gamma_2\Delta^2\Psi$	$N(\Psi+\gamma_2\Delta^2\Psi)$	Observed frequency
0.....	0.4890	+0.4890	+0.4890	567.73	+220.37	788.1	751
1.....	0.3495	−0.1395	−0.6285	405.77	−283.23	122.5+	179
2.....	0.1252	−0.2243	−0.0848	145.54	− 38.21	107.3	107
3.....	0.0300	−0.0952	+0.1291	34.83	+ 58.50	93.3	69
4.....	0.0054	−0.0246	+0.0706	6.27	+ 31.82	38.1	46
5.....	0.0008	−0.0046	+0.0200	0.93	+ 9.01	9.9	7
6.....	0.0001	−0.0007	+0.0039	0.12	+ 1.76	1.9	2
7.....	0.0000	−0.0001	+0.0006	0.00	+ 0.27	0.3	0

TABLE VIII

x	ξ	$\Psi(\xi)$	$\Delta\Psi$	$\Delta^2\Psi$	$N\Psi$	$N\gamma_2\Delta^2\Psi$	$N(\Psi+\gamma_2\Delta^2\Psi$	Observed frequency
10.....	0	0.7246	+0.7246	+0.7246	1,805.70	+298.03	2,103.7	2,089
9.....	1	0.2328	−0.4918	−1.2164	580.14	−500.31	79.8	117
8.....	2	0.0380	−0.1948	+0.2970	94.70	+122.16	216.9	194
7.....	3	0.0042	−0.0338	+0.1610	10.47	+ 66.22	76.7	71
6.....	4	0.0004	−0.0038	+0.0300	1.00	+ 12.34	13.3	19
5.....	5	0.0000	−0.0004	+0.0034	0.00	+ 1.40	1.4	2
4.....	6	0.0000	0.0000	+0.0004	0.00	+ 0.16	0.2	0

TABLE IX

Cloudiness x	Ordinate of 1st Charlier curve y_1	Ordinate of 2d Charlier curve y_2	Sum of two Charlier components y_1+y_2	Observed frequency	Area of Pearson curve	1st Charlier approximation
0.....	788.1	.....	788.1	751	803	754.3
1.....	122.5+	.....	122.5+	179	142	168.8
2.....	107.3	.....	107.3	107	72	113.0
3.....	93.3	.....	93.3	69	60	80.2
4.....	38.1	0.2	38.3	46	51	30.8
5.....	9.9	1.4	11.3	9	50	9.3
6.....	1.9	13.3	15.2	21	55	16.7
7.....	0.3	76.7	77.0	71	60	85.2
8.....	.....	216.9	216.9	194	85	228.6
9.....	.....	79.8	79.8	117	153	46.4
10.....	.....	2,103.7	2,103.7	2,089	2,122	2,119.7
Total.....	.....	.....	3,653.4	3,653	3,653	3,652.9

however, is attracted to the peculiar discrepancy in the fit at degree 9. This discrepancy may be due to faulty observation, for—to quote Pearson [1]—"as Herr Meyer remarks, the observer must have had some personal bias with regard to the grade 9." In addition to the fact that the composite Charlier curve seems to afford a better fit, the

[1] Pearson, "Cloudiness," p. 287. Pearson's reference is, of course, to Meyer, *loc. cit.*

amount of computation required in obtaining it is decidedly less than that demanded by Pearson's method. It would be considerably less even if areas had not been used in obtaining Pearson's theoretical frequencies.

> It must be borne in mind that any series involving derivatives or finite difference can theoretically be used on a distribution of a continuous variable only by resorting to mechanical quadrature. . . . This applies to Charlier's type B series as well as to Pearson's generalized curves. Practically, such series are, however, entirely satisfactory since the distribution may be treated as one of a discrete variable by leaving all moments unmodified and permitting the ordinates, instead of areas, to represent the graduated frequencies.[1]

In conclusion it may be remarked that the possibility of fitting a U-shaped distribution with a sum of two Charlier curves raises the question whether the variations involved in the series arise from conditions that can be separated into two distinct parts.

[1] Carver, *loc. cit.*, p. 119.

NOTES

PROBABLE INFERENCE, THE LAW OF SUCCESSION, AND STATISTICAL INFERENCE

By Edwin B. Wilson, *Harvard School of Public Health*

Probable Inference (Usual). If there be observed a certain frequency or rate p_0 in a population of n and if the corresponding standard deviation $(p_0q_0/n)^{1/2}=\sigma_0$ be computed, the common statement of probable inference is to say that: The probability that the true value of the rate p lies outside its limits $p_0-\lambda\sigma_0$ and $p_0+\lambda\sigma_0$ is less than or equal to P_λ. It is assumed that P_λ decreases with an increase of λ. If the criterion of Tchebycheff is used, P_λ is itself less than $1/\lambda^2$; but if the probability table is used, P_λ is the area under the probability curve beyond the ordinates $\pm\lambda\sigma_0$. The rule of Tchebycheff is exceedingly conservative in its estimate of P_λ, whereas the probability table gives a radical estimate.

Strictly speaking, the usual statement of probable inference as given above is elliptical. Really the chance that the true probability p lies outside a specified range is either 0 or 1; for p actually lies within that range or does not. It is the observed rate p_0 which has a greater or less chance of lying within a certain interval of the true rate p. If the observer has had the hard luck to have observed a relatively rare event and to have based his inference thereon, he may be fairly wide of the mark.

Probable Inference (Improved). A better way to proceed is to reason as follows: There is some rate p. Its standard deviation is $(pq/n)^{1/2}=\sigma$. The probability that an observation as bad as p_0 will occur, where p_0 lies outside the limits $p-\lambda\sigma$ and $p+\lambda\sigma$, is less than or equal to P_λ. This form of statement throws the emphasis upon the fallibility of a particular observation in respect to being typical of a general situation.

It is still possible to state the criterion in terms of the observed rate p_0 for the equation $(p_0-p)^2=\lambda^2pq/n$, where $q=1-p$, is quadratic in p and may be solved to find p. If $\lambda^2/n=t$, the solution is

$$p=\frac{p_0+t/2}{1+t}\pm\frac{\sqrt{p_0q_0t+t^2/4}}{1+t}.$$

The rule then may be stated as: If the true value of the probability p lies outside the range

$$\frac{p_0+t/2}{1+t}-\frac{\sqrt{p_0q_0t+t^2/4}}{1+t} \quad \text{and} \quad \frac{p_0+t/2}{1+t}+\frac{\sqrt{p_0q_0t+t^2/4}}{1+t},$$

the chance of having such hard luck as to have made an observation so bad as p_0 is less or equal to P_λ. And this form of statement is not elliptical. It is the proper form of probable inference.

Concerning the range indicated, it may be remarked that it is not centered at the value p_0 but at the value $(p_0+t/2)/(1+t)$ which differs from p_0 by being displaced toward the value 1/2 by the amount

$$\frac{p_0+t/2}{1+t}-p_0=\frac{t(1/2-p_0)}{1+t}=\frac{(q_0-p_0)t/2}{1+t}.$$

Moreover, the interval on either side of the mean is

$$R=\sqrt{p_0q_0/n+\lambda^2/4n^2}/(1+\lambda^2/n),$$

which is not identical with $\lambda\sigma_0$ computed from p_0 nor with that value $\lambda\sigma_c$ which might be computed from the central value $(p_0+t/2)/(1+t)$ of the range indicated. In fact $R<\lambda\sigma_c$ and $\lambda\sigma_0<\lambda\sigma_c$, but R may be either less than or greater than $\lambda\sigma_0$—less if p_0 lies between .067 and .933, greater if p_0 lies outside those limits unless $t=\lambda^2/n$ be considerable compared with 2. The precise lines of division are $p_0=\frac{1}{2}\pm\frac{1}{2}\sqrt{1-(2+t)^{-2}}$.

The Law of Succession. The law of succession of Laplace states that if we have experienced S successes and F failures out of $S+F=n$ trials, the chance of success on the $(n+1)$st trial is $(S+1)/(n+2)$. Thus the law of succession purports to give the probability from experience not as $p_0=S/n$ but as $p=(S+1)/(n+2)$. This chance is, however, not the true chance of success, because the chance of success p on every trial must be the same. The proof of the law depends on inverse probabilities and in particular on the assumption that all probabilities are *a priori* equi-probable. The proof has been much criticized, for it has been held that the experience $p_0=S/n$ does not permit the assumption that all probabilities are equi-probable, but indicates that those in the neighborhood of p_0 must be much more probable than those remote from p_0. The simplest, if crudest, form of the argument of equi-probability is found in interpreting the formula $(S+1)/(n+2)$ as giving two new trials of which one is assumed to be a success and the other a failure.

If we apply the criterion in terms of the standard deviation as above developed we may state that the center of the range for p is $(p_0+t/2)/(1+t)$. If we now replace t by λ^2/n, and $p_0 n$ by S, the center of the range becomes $(S+\lambda^2/2)/(n+\lambda^2)$, and the probable inference is this: If the true probability lies outside the range

$$\frac{S+\lambda^2/2}{n+\lambda^2}-\lambda\frac{\sqrt{SF/n+\lambda^2/4}}{n+\lambda^2} \text{ and } \frac{S+\lambda^2/2}{n+\lambda^2}+\lambda\frac{\sqrt{SF/n+\lambda^2/4}}{n+\lambda^2},$$

the chance of our having the hard luck to realize the observed value $p_0=S/n$ is less than or equal to P_λ. As the distribution of the chances of an observation is asymmetric, it is perhaps unfair to take the central value of the range as the best estimate of the true probability; but this is what is actually done in practice.

In terms, therefore, of the practical criterion the forecasted value of the true probability is

$$\text{not } \frac{S+1}{n+2}, \text{ nor } \frac{S}{n}, \text{ but } \frac{S+\lambda^2/2}{n+\lambda^2};$$

and the value that should be assigned depends on the value of λ, *i. e.*, on our readiness to gamble on the typicalness of our realized experience. From this viewpoint, only those who believe that their experience is absolutely typical will set $\lambda=0$ and use as a forecast the realized frequency S/n. Those who use the law of succession, set $\lambda^2=2$ and allow a total variation in their experience of 2.8σ, *i. e.*, they wish to assert that they have not had an experience so rare that it or one less probable would arise, on the basis of the probability table as an estimate of P_λ, less than 16 times in 100. Those who make the usual allowance of 2σ for drawing an inference would use $(S+2)/(n+4)$ as a law of succession.

A particularly interesting and instructive case is that in which there has been total failure, $p_0=0$, $\sigma_0=0$. Here clearly the first form of the inference, namely, that the true value of p must lie between $p_0-\lambda\sigma_0=0$ and $p_0+\lambda\sigma_0=0$ is out of the question. The true form states that the experience is not so unusual as P_λ if p is less than $\lambda^2/(n+\lambda^2)n$ or if the expected number of instances is less than $\lambda^2/(n+\lambda^2)$, which for n large is practically λ^2/n. If this were applied to the classic case of determining the chance that the sun should fail to rise, one would take λ very small compared to 1 because general considerations of astronomy make it highly probable that our past experience is very nearly typical. If the application were to the fact that there were no deaths from leprosy in Massachusetts ($n=4{,}000{,}000$) in 1924, λ would also be taken small because leprosy is so rare, perhaps $\lambda=2/3$, meaning that we would take

an even chance. But in the case of paratyphoid fever, we might prefer to use the ordinary criterion with $\lambda = 2$.

Statistical Inference. This brings us to statistical inference which had best be differentiated from probable inference by requiring that something over and above the value of p_0 be known, something that will motivate a choice among values for λ in drawing the inference. It is well known that some phenomena show less and some show more variation than that due to chance as determined by the Bernoulli expansion $(p+q)^n$. The value L of the Lexian ratio is precisely the ratio of the observed dispersion to the value of $(npq)^{1/2}$ or $(pq/n)^{1/2}$ as the case may be. If we have general information which leads us to believe that the variation of a particular phenomenon be supernormal $(L>1)$, we naturally shall allow for some value of L in drawing the inference. Thus if the Lexian ratio is presumed from previous analysis of similar phenomena to be in the neighborhood of 5, we may use $\lambda = 10$ as properly as we should use $\lambda = 2$ if the phenomenon were believed to be normal (Bernoullian).

MEASUREMENT OF VARIATIONS IN SEASONAL DISTRIBUTION

By J. C. Clendenin

During the past three or four years, sporadic discussions as to the possibility and value of measuring shifts or variations in seasonal tendencies have appeared among statisticians interested in time series. In almost every case the fact that such variations might take place has been freely admitted, but there has been debate upon two questions: first, whether such shifts occur frequently enough and are of such magnitude as to demand attention, and second, whether the available methods of measuring them are accurate enough and economical enough to warrant their use. It is the purpose of the present study to answer the first question, and, in response to the second, to present a method for measuring shifting seasonal tendencies which is both accurate and simple.

In order to attack the first question in logical fashion, it is necessary to point out the uses to which the seasonal indices are put. These may be classified under two heads: first, they are used as factors in the analysis of the data, to help determine causes and relationships; and secondly, they are used as elements in synthesis, as developing factors in forecasting and planning for the future. As analytical factors, they help us to isolate and measure the quantitative effects of causal elements—the business cycle, competitive pressure, advertising campaigns, fad and fashion, or what-not—by eliminating from the analysis the effects of ordinary seasonal tendencies. Incidentally, it is the opinion of the writer that surprising results will some day be achieved in the further refinement of these seasonal indices, by examining their regression upon some of the other factors just indicated. In the synthetic process of forecasting and budget making the proper seasonal measurements, combined with the results achieved by analyzing the past, make possible the accurate and economical directing of the future—a consummation devoutly to be wished.

Everyone is familiar with the objectives which have drawn the attention of time series statisticians in the past. The business cycle in the United States and in the world has been closely examined, and its causes and characteristics broadcast. The progress of our major industries has been likewise subjected to scrutiny, and at present it is possible to predict the general progress of business and its great indus-

tries with considerable accuracy. It is highly significant to note, in observing this group of very gratifying results, that the measurement of shifting seasonal indices has not been necessary. Indeed, the most searching scrutiny fails to show seasonal shifts in most of the generalized indices, such as bank clearings or freight car loadings; and it is only in certain individual industries which are undergoing change, that the shifts appear. The seasonal distribution in the automobile industry changed, for example, when the closed car made winter sales possible, and the sales of electric power have shown a similar response to the development of electric factory machinery and the introduction of fans, sweepers, stoves, and refrigerators for the home. Such is the experience of the past, but the problem of the present is vastly different.

Since the close of the post-War depression of 1921–22, increasing attention has been given to the analysis of time series showing (1) the progress of localized communities, and (2) the progress of specific industries and individual concerns. Though previously recognized, the fact that certain communities can run counter to their neighbors has not heretofore been given due emphasis. Simultaneous Florida boom, Iowa depression, and general fair prosperity have been recognized in the establishment of numerous economic and statistical organizations reporting upon local conditions, and naturally drawing upon local sources for informative data. The advantages to be achieved by analysis of production and sales data in individual concerns are not being overlooked, either, having become one of the important elements in modern competition. Demands of this type, and similar demands for analysis of each separate industry, have necessitated the use of further specialized data, data which often lacks the balance and steadiness imparted to generalized national indices by the contributions of many industries and communities. It is in the handling of data from localized and specialized sources that the need for the measurement of variations in seasonal movements is likely to manifest itself.

The writer has had occasion to study many communities and industries on the Pacific Coast, in connection with the researches of an organization [1] reporting upon business conditions in several regional areas, where in many cases the measurement of changes in seasonal movement was absolutely essential. These Coast communities have grown very rapidly, changing the nature and scope of their industries and their trading habits almost year by year. Obviously, the industries dependent upon them have changed also, and have perhaps initiated changes of their own. In regional car loadings and bank clearings data we have occasionally observed shifts of 10 per cent per year

[1] Silberling-Schaffer Business Reports, 2163 Center, Berkeley, California.

in the seasonal percentages, while changes of 5 or 3 per cent have been common. While we are perfectly cognizant of the fact that seasonal indices cannot be determined with precise accuracy, such movements as we have found are unmistakable, as will be noted in examining the accompanying diagrams and table. Our conclusion is, therefore, that the measurement of shifts or variations in the seasonal distribution is often necessary in studying data of individual concerns, specific industries, and localized areas, and that tests are justified under almost any condition. Several years' data are necessary to establish the presumption of a normal, in order to measure seasonal movements at all; and in that period of time those seasonal movements may have undergone extensive change.

The practical statistician has in general three criteria by which he judges any method of deriving an index of seasonal variation. First of all, he is interested in its accuracy. It must be as free as possible from mechanical pitfalls, it must be capable of caring for freakish data without absorbing any vagaries, it must neither conceal nor cumulate error, and it should be mathematical enough to avoid burdening the operator with too many questions for judgment. Secondly, the calculations required must be simple and short, as economy of time and effort is often of paramount importance. Thirdly, the completed index must be readily applicable to the problem in hand. For this last reason, the indices calculated by all of the common methods state the seasonal factor as a percentage of the corresponding item of trend—that is, a January seasonal percentage of 110 means that the normal January is 110 per cent of the January ordinate of trend.

In connection with this latter point, it is interesting to notice a gross error which is made in the computation of the ordinary seasonal indices. As we have seen, the seasonal indices state each month (assuming monthly data for illustrative purposes) as normally a given per cent of the corresponding monthly ordinate of trend. The trend line itself is usually calculated to describe the general progress of the series, and in so doing indicates the absolute normal which the series would have followed had no seasonal, cyclical, or random influences entered in. Thus in any *normal* year the volume of business expected would be shown by the total of the twelve monthly trend ordinates. The function of the normal seasonal factors is therefore to help to distribute that total throughout the year, but never to modify it. But the last step in each of the common methods of computing seasonal indices is to adjust the year's indices proportionally until they average 100 per cent—or to adjust twelve monthly seasonal items to a total of 1,200. What justification is there for asserting that such an

adjustment will assure the desired maintenance of the same total in applying the seasonal corrections to the trend ordinates, when we know that the twelve percentages are applied to twelve different trend ordinates? Except when the trend is known to be such that the actual percentage deviations about it total zero, the method is in error, often seriously so. The writer has never seen a trend calculated in actual practice which meets this test. Instead of the ordinary adjustment, the more accurate plan would be to apply the unadjusted seasonal factors to their respective trend ordinates within one representative year, and then adjust the seasonal indices proportionally to eliminate the discrepancy between the total of the trend ordinates and that of the seasonally modified items. This trial and error process requires some additional work, but it would eliminate frequent serious errors in the handling of time series.

But to return to the measurement of varying seasonal distributions —two methods have been generally discussed. The first of those is based upon Professor Persons' ingenious device, link relatives. Instead of merely selecting one typical group of relatives to represent the entire period in question, all of the relatives for January are arrayed in chronological order and their general progress described by curve or moving average. This is done for each month (or week, quarter, etc.), and a separate set of "smoothed" relatives thus obtained for each year. Each set must then be put through the chain multiplication, trend correction, and final adjustment, to obtain a separate set of seasonal indices for each year. Most of the advantages of the ordinary link-relative plan adhere to this one also. The results are fairly accurate, but exceedingly costly, due to the amount of labor involved.

The second plan[1] which requires notice is that proposed by Dr. O. Gressens, of the University of Illinois. Dr. Gressens attacks the problem very directly, basing his computations upon the ratios of the actually occurring items to the corresponding trend ordinates. The development of the ratio is followed exactly as is done under the link-relative plan, graphically or by a moving average or moving median. The resulting "smoothed" ratios show the actual normal development, cyclical and random factors having been eliminated by the smoothing process. The smoothed ratios can then be collected into separate sets for each year, and adjusted to an average of 100 per cent to complete the indices. The method yields very acceptable measurements of any major shift in the seasonal distribution if the data extend over a long period of time. For periods of ten years or less, however, the movements due to the business cycle and to random fluctuations

[1] This JOURNAL, June, 1925.

are often so pronounced in the ratios that accurate smoothing is impossible. Under such conditions the method is unsatisfactory. This disadvantage might be overcome by using ratios of the actual items to a twelve months' moving average, centered, instead of ratios of actual to trend, but the additional cost would be noticeable.

Since the so-called "smoothing" process is the key to the accuracy of all the methods of measuring a shifting seasonal distribution, it must be given careful notice. Three plans have been suggested to accomplish the smoothing—the use of a moving average, a moving median, or a fitted line. The smoothing process must eliminate cyclical and random tendencies, but should not disturb the purely seasonal elements, which may be constant, progressively shifting, or which may make sudden and radical changes occasionally. Though abrupt shifts do take place they are not common, and where there is any doubt the operator is justified in assuming the change to be a gradual one. Much depends upon the judgment of the operator, and for this reason it is usually preferable to chart the ratios, that their progress may be more readily seen. The best method of smoothing would be the fitting of "trends" to the ratios, but as this is a quite burdensome process, freehand curves may often be used, supplemented by an average or two as a guide. The moving average is usually satisfactory, also, but the moving median is scarcely dependable enough.

It is apparent that the plans thus far discussed have all been very laborious, and have all depended to some extent upon the judgment of the operator. The latter defect seems unavoidable, but it is possible to eliminate some of the work by using a third method, which is presented as the outcome of experiments in handling the Pacific Coast data previously mentioned.

Briefly, the theory underlying our method is this: instead of measuring actual seasonal deviations from the normal trend, we measure the normal relative values of the different months (or weeks, quarters, etc.) of the year, stating each value as a per cent of the year's average. Then the problem of secular trend becomes that of determining the progress of the average month of the year under normal conditions—a matter of calculating the trend of the *average* monthly value, to determine one single ordinate of trend to represent each year. Our "seasonal" indices may then be applied to this single ordinate of trend, to determine positive monthly normals from which cyclical and irregular deviations may be measured.

The method of calculating the "seasonal" indices under this plan may be described as follows: (1) Each monthly item for each year is

stated as a per cent of the average of the twelve monthly items of that particular year. (2) The percentages for similar months may next be collected, ranged in chronological order, and the development of the percentage value for each month determined by a smoothing process, as previously suggested. The accompanying diagram and table illustrate the determination of the "smoothed" ratios for three different months. (3) Collect the twelve smoothed ratios for each year, and adjust them proportionally to a total of 1,200. It will be noted that the "adjustment to 1,200" is mathematically correct in this application, as the percentages are upon a common base, the single trend value for the year.

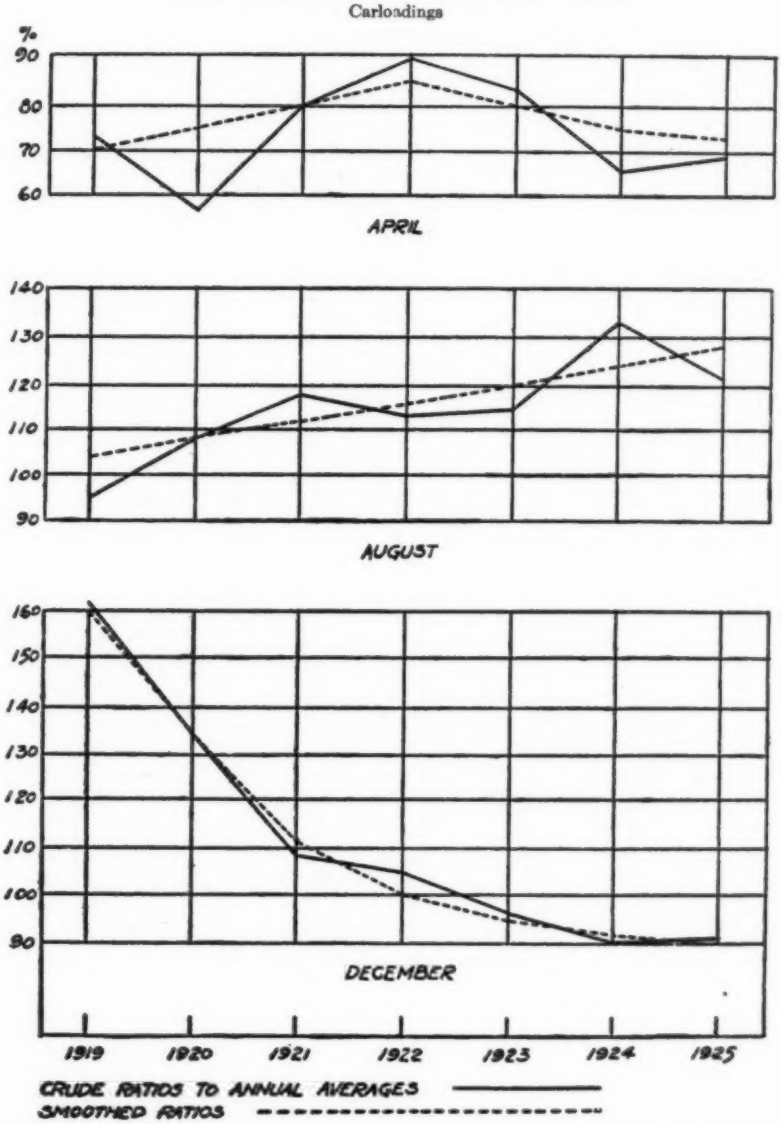
It is apparent that the "indices" computed by the above method contain two elements—the true seasonal factors, which cause the deviation of the trend-seasonal normals from the trend line itself, and the within-the-year trend growth. By combining these two elements into the seasonal index, we make it possible to calculate normals, make forecasts, and complete cyclical analyses with but one trend ordinate needed for each year. This fact is in itself a time saver. In addition, the basic ratios are all calculated with a common denominator for each year, and in computing normals each seasonal factor within the year is multiplied by the same trend ordinate. Whether the computations are made by slide rule or calculating machine, common multipliers and divisors save time and are conducive to accuracy.

The greatest disadvantage of this method is that the indices do not make possible the elimination of seasonal movements alone from a series. Trend and seasonal may be removed together, and almost every other need of the time series analyist is served, but the purely seasonal factor remains unmeasured. It would be possible to devise a further step in the process of computation, by which the within-the-year trend growth could be removed from the indices, but the writer does not believe that it is either economical or necessary to do so.

The accompanying examples are drawn from the carloadings data of the leading railroad in an important Pacific Coast area. Only seven years' data were available at the time the study was made, yet the effects of a changing industrial and commercial make-up are clearly apparent. Since the basic ratios are influenced only in a minor way by the business cycle (it seems impossible for more than two successive Januaries, for example, to be cyclically high or low), it is not at all impossible to trace the development of the ratios accurately. August and December present no problem at all; April is unusually difficult, but observation of neighboring months during 1921, 1922, and 1923 seems to indicate a passing tendency during those three

years to concentrate certain shipping activities in the spring. Apparently there was no cyclical element involved, or it was not of great importance. The accuracy of the work as finally completed is attested by the smallness of the adjustments necessary to bring the twelve indices for each year to a total of 1,200. Obviously, perfection has

COMPUTATION OF SHIFTING SEASONAL VALUES
Carloadings



not been reached, but we believe that we have achieved a very fair degree of accuracy with by far the least laborious method which has yet been suggested for caring for a shifting seasonal distribution.

COMPUTATION OF SHIFTING SEASONAL VALUES

Carloadings

	April			August			December		
	Ratios to annual averages	Smoothed seasonal values	Adjusted to total of 1,200*	Ratios to annual averages	Smoothed seasonal values	Adjusted to total of 1,200*	Ratios to annual averages	Smoothed seasonal values	Adjusted to total of 1,200*
1919.....	73	70	69	95	104	103	162	160	158
1920.....	56	75	74	108	108	107	134	134	133
1921.....	80	80	80	118	112	111	108	112	111
1922.....	89	85	85	113	116	115	105	100	100
1923.....	83	80	80	115	120	120	96	95	95
1924.....	66	75	75	133	124	124	90	92	92
1925.....	69	73	73	122	128	128	91	90	90

* Completed indices.

REPORT TO DATE OF THE WEEKLY INDEX NUMBER

By Irving Fisher

For 1927 the base week used is the last week in December, 1926. The indexes of this common week are equated in both the 1926 and 1927 series, to maintain continuity, at the figure 146.1.

The number of commodities in the 1927 series is 198. The list is identical with the 1926 list except for the dropping of four commodities: "Woolen Goods, 36-inch cotton-warp serge," no longer on the market, and three minor commodities quoted so irregularly that they also have been dropped, "Hides, No. 1 Kip," "Hides, No. 1 Calfskin" and "Crude Corn Oil."

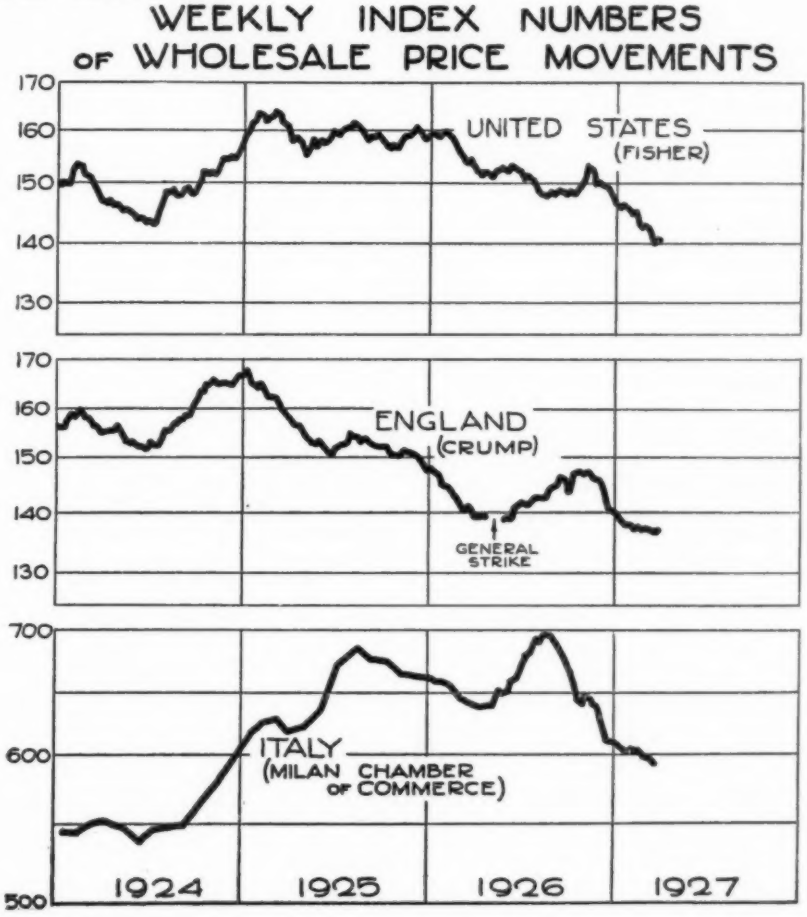


TABLE I
WEEKLY INDEX NUMBERS
(Continuation of Table I, this JOURNAL, September, 1926, p. 344)

For week ending Friday	Index number of prices in per cent of pre-war, i. e., 1913=100 per cent	Purchasing power of the dollar in pre-war cents. i. e., 1913=100 cents
1926 (*Cont.*)		
July 2	150.2	66.6
July 9	150.8	66.3
July 16	150.2	66.6
July 23	149.0	67.1
July 30	148.0	67.6
August 6	147.7	67.7
August 13	147.3	67.9
August 20	147.2	68.0
August 27	148.1	67.5
September 3	147.6	67.8
September 10	148.5	67.4
September 17	148.3	67.4
September 24	147.8	67.7
October 1	148.3	67.4
October 8	147.4	67.8
October 15	148.1	67.5
October 22	148.4	67.4
October 29	150.0	66.7
November 5	153.5	65.2
November 12	152.9	65.4
November 19	149.2	67.0
November 26	149.4	66.9
December 3	149.5	66.9
December 10	149.0	67.1
December 17	147.2	67.9
December 24	146.5	68.3
December 31	146.1	68.5
1927		
January 7	145.9	68.6
January 14	145.1	68.9
January 21	146.0	68.5
January 28	145.0	69.0
February 4	144.0	69.4
February 11	144.4	69.3
February 18	142.1	70.4
February 25	141.4	70.7
March 4	142.1	70.4
March 11	141.4	70.7
March 18	139.6	71.7
March 25	140.4	71.2
April 1	140.3	71.3
April 8	140.9	71.0
April 15	140.1	71.4
April 22	139.5	71.7
April 29	139.6	71.6
May 6	140.1	71.4
May 13	139.6	71.6

TABLE II
MONTHLY AVERAGES
(Continuation of Table II, this JOURNAL, September, 1926, p. 345)

	Index number 1913=100	Purchasing power 1913=100
1926 (*Cont.*)		
June	152.0	65.8
July	149.5	66.9
August	147.6	67.8
September	148.2	67.5
October	148.5	67.4
November	150.9	66.3
December	147.2	67.9
1927		
January	145.5	68.7
February	143.0	69.9
March	140.7	71.1
April	140.0	71.4

TABLE III

QUARTERLY AVERAGES

	Index number 1913=100	Purchasing power 1913=100
1923		
1st Quarter	161.3	62.0
2nd Quarter	162.6	61.5
3rd Quarter	154.2	64.9
4th Quarter	152.9	65.4
1924		
1st Quarter	150.5	66.4
2nd Quarter	144.7	69.1
3rd Quarter	147.3	67.9
4th Quarter	153.9	65.0
1925		
1st Quarter	161.9	61.8
2nd Quarter	157.2	63.6
3rd Quarter	159.2	62.8
4th Quarter	158.3	63.2
1926		
1st Quarter	156.3	64.0
2nd Quarter	151.7	65.9
3d Quarter	148.4	67.4
4th Quarter	149.0	67.1
1927		
1st Quarter	142.9	70.0

TABLE IV

YEARLY AVERAGES

	Index number 1913=100	Purchasing power 1913=100
1923	157.7	63.4
1924	149.3	67.0
1925	159.2	62.8
1926	151.3	66.1

CLASSIFICATION OF MEMBERS OF AMERICAN STATISTICAL ASSOCIATION ON BASIS OF DUTIES AND INTERESTS

By Willford I. King

A year or more ago, the Secretary of the American Statistical Association sent out to the members, blanks upon which they were requested to give information concerning the type of work in which they were engaged and the nature of their statistical interests. Six hundred and twenty-nine of the members filled out these forms completely enough so that the results were susceptible of tabulation.

Mr. Elwood J. Way of the Statistical Tabulation Bureau, 1301 H Street, N. W., Washington, D. C., felt that the members of the Association would be interested in having this matter tabulated properly. Since his Bureau is equipped with Hollerith machines, he felt that he could conveniently analyze the results by placing the data on punch cards and running the cards through the machines. He volunteered to do this work for the American Statistical Association without cost to our Organization. This favor was gratefully accepted and the Statistical Tabulation Bureau completed the work of analysis and delivered to us the completed tables.

Mr. Way tabulated the data by states as well as by sections of the country but, inasmuch as in a number of states too few members made returns to secure statistical regularity, it appears that the totals for the various sections of the United States express more satisfactorily the conditions prevailing than do the items for the individual states. The fact should be noted that Mr. Way has included Delaware, Maryland, and the District of Columbia with the South Atlantic group. Most of our members in that section of the United States are located in and around Washington, D. C.; hence they dominate the results for the South Atlantic group.

We see from Table I that, of the members reporting, approximately one-half class themselves as Research Workers, about one-fourth as Executives and about one-sixth as Teachers. The proportions differ less widely in different sections of the country than one might expect. However, it may be noted that Teachers play a larger rôle in New England than in most of the other sections and occupy a relatively unimportant place in the South Atlantic group. This situation is due

TABLE I

CLASSIFICATION OF 629 REPORTING MEMBERS OF THE AMERICAN STATISTICAL ASSOCIATION ACCORDING TO SEX, DUTIES, AND ORGANIZATION BY WHICH EMPLOYED

	Section of United States									
	All sections	New England	Middle Atlantic*	East North Central	West North Central	South Atlantic*	East South Central	West South Central	Mountain	Pacific
Total Number	629	75	288	117	22	84	6	8	8	21
Male	568	69	253	108	21	76	6	8	8	19
Female	61	6	35	9	1	8	..	..	..	2
Duties:										
Executive	178	12	94	33	7	24	2	3	..	3
Research worker	310	27	151	54	6	52	2	5	3	10
Teacher	119	30	37	27	9	4	2	..	3	7
All others	22	6	6	3	..	4	..	..	2	1
Employed by:										
Federal and state government	137	5	28	26	6	55	4	4	2	7
Municipal government	132	34	52	21	5	9	..	..	6	5
Public service corporation	53	1	38	7	1	3	..	..	..	3
Manufacturing establishment	62	9	28	22	3	..	..	..	..	..
Mercantile establishment	19	1	12	2	2	..	..	2	..	..
Trade association	26	1	10	13	..	2	..	..	..	..
Financial institution	42	3	31	4	1	..	..	..	..	3
Professional service organization	53	5	34	7	..	4	..	..	..	3
Foundations, etc.	36	2	20	2	2	8	2	..	..	..
Insurance	52	11	25	10	2	2	..	2	..	..
Publishers and printers	17	3	10	3	..	1	..	..	..	..

* Delaware, Maryland, and the District of Columbia are included with the South Atlantic group.

to the fact that, in the South Atlantic group, a large proportion of the members are Washingtonians employed by the Federal Government.

Table II shows the relative degree of interest in different sections of the country in different types of statistical work. The figures indicate clearly that Business Facts are of less interest to the members in the South than in other sections of the country. The West and South on the other hand are more interested in Vital Statistics than are the North and East. Some one-fifth to one-fourth of all the members, without much regard to where they happen to live, are interested in Economic Problems. Finance has the strongest appeal in the Northeast and on the Pacific Coast. The West South Central division shows the largest percentage of interest in Statistical Method and the Pacific division the least.

Taking the country as a whole, by far the greatest interest is in Business Facts, with Economic Problems coming next. Vital Statistics and Statistical Method are next in point of importance. Although this is a statistical association, interest in Statistical Method is only about one-half as great as in Economic Problems, and one-third as

great as in Business Facts. Our members ascribe more importance to Finance than to Social Welfare and Education combined but they are not as much interested in Finance as in Statistical Method.

The thanks of the Association are certainly due to Mr. Way and the Statistical Tabulation Bureau for this careful analysis.

TABLE II

RELATIVE DEGREES OF INTEREST IN VARIOUS PHASES OF STATISTICAL WORK EXPRESSED BY MEMBERS FILING REPORTS

	Per Cent In Each Section of the United States									
	All sections	New England	Middle Atlantic*	East North Central	West North Central	South Atlantic*	East South Central	West South Central	Mountain	Pacific
All Interests	100	100	100	100	100	100	100	100	100	100
Business facts	38	39	39	44	36	25	27	21	37	33
Vital statistics	14	13	13	14	18	20	20	11	17	19
Economic problems	21	21	20	17	19	31	7	26	26	25
Finance	9	10	10	6	7	7	7	5	3	10
Social welfare	5	3	5	5	4	4	13	...	..	2
Education	2	3	1	2	3	3	13	5	..	3
Statistical method	11	12	11	11	14	11	13	32	17	8

* Delaware, Maryland and the District of Columbia are included with the South Atlantic group.

ANSWER TO THE REVIEW OF "AMERICAN VILLAGERS"

To the Editor of the Journal of the American Statistical Association:

Sir:

Mr. Thompson's review of *American Villagers*, which appeared in the March, 1927, issue of this JOURNAL, contains misstatements of fact. His review begins with the assertion that "there is nothing new in the part of this book describing these villages." The fact is that the data upon the socio-economic status of the gainfully employed workers and all of the material about the number of village doctors, lawyers, ministers, nurses, etc., are entirely new and did not appear in my previously published pamphlets to which he refers. With one or two minor exceptions, the only parts of the book that are not entirely new are the materials in Chapter IV, and part of Chapter V, and even here the data are presented in a new form.

Mr. Thompson is chiefly distressed by the failure of this book (which deals with villages) to use the farm population census. He says: "One can scarcely believe that Dr. Fry and Dr. Cressman are both ignorant of its existence, and yet no other reason can be assigned for the failure to use it." The farm population census is specifically mentioned on page 22 of *American Villagers*, and in my earlier pamphlets, of which Mr. Thompson considers the present volume only a rehash, comparisons between village and farm population figures are presented in detail.

As glaring examples of the omission of farm population figures, Mr. Thompson points to Chapters II and III and to the Appendix. The title of Chapter II is "How Many Villages Are There?" Just why a discussion of farm population is necessary to answer this question I do not see. Chapter III deals with the question, "Are Village Populations Declining?" As this part of the analysis is confined to a comparison of census data for 1900, 1910, and 1920, and as farm population figures are available for 1920 only, I do not see any basis upon which farm figures could have been introduced.

Regarding the Appendix, Mr. Cressman, who was responsible for it, makes the following comment:

"We not only knew that Mr. Truesdell was making a study of the rural population for the Bureau of the Census but some advance sheets of population totals were kindly made available by him as checking material for the data in the Appendix. This material of Mr. Truesdell showed the accuracy of the estimates of population presented in the Appendix. Since no actual use was made of these advance sheets in the study and since the author was specifically requested not to use any of the data because they had not yet been made public, no reference was made to them in the article."

Mr. Thompson cannot understand why open-country population used in *American Villagers* is 39 million when the census shows the country's total farm population to be slightly less than 32 million. The point is that, by very definition, they comprise fundamentally different groups. The surprising thing is that Mr. Thompson should imply any need for agreement between them.

I hardly think any further comment is necessary.

Faithfully yours,

C. LUTHER FRY

COMMENT ON THE REVIEW OF "BUSINESS ANNALS"

To the Editor of the Journal of the American Statistical Association:

Sir:

In the review of *Business Annals* which appeared in the last number of this JOURNAL, Dr. Hardy criticized the record given of the Brazilian coffee industry. Further study of these records, both by Dr. Hardy and myself, has led to our mutual recognition of the fact that Brazilian coffee records are not only confused, but that the most reputable authorities are often in disagreement. Because I fear that the review may lead some persons to feel that the volume as a whole is unreliable, may I comment briefly on the points which it raised?

1. Dr. Hardy's objection to the description of the coffee price in 1901 as rising, is based on the fact that 1901 prices did not reach the level of the years immediately previous, high because of the "bubonic-plague boom." The *Economist*, the *Statist*, and William H. Ukers, editor of the *Tea and Coffee Trade Journal*, show a rise of prices within the year, with the low point in April.

2. Dr. Hardy states that the crop of 1903, described in the *Annals* as "good," was really "the smallest, with one exception, between 1899 and 1918." There were actually four smaller crops between 1899 and 1918. Dr. Hardy was misled by a series which he kindly sent me, and which I find upon investigation, cover only Rio and Santos coffee to 1904, and Rio, Santos, Victoria and Bahia from 1905 to date. The figures generally accepted, and confirmed by the Brazilian Consul-General in New York City, for the four ports for the entire period, show the statement in the *Annals* to be correct. In fact, only four previous coffee crops had been larger.

3. In regard to the records for the years 1919 and 1920, I was misled by an ambiguous table in the 1922 *Annuaire Statistique*, in which, under the heading of production, export figures were given since 1914. In the 1924 edition, not then available, footnotes are added which would have prevented such an error.

4. The *Annals* should have stated that the frost of 1918 "rendered useless" one-half of the coffee trees rather than "destroyed" them. *Miscellaneous Bulletin 86*, United States Bureau of Foreign and Domestic Commerce, the authority used, states that "The number of bearing bushes in Brazil was estimated at about 790,000,000, of which about 350,000,000 were rendered useless by the frost as well as many of the young bushes not yet bearing coffee." Furthermore, the 1918 crop was 61 per cent, and the 1919 crop, 47 per cent of that for 1917.

5. The fact that the 1905 crop had a high gold value (which had been exceeded three times in the previous ten years instead of "once or twice previously," as Dr. Hardy states) is not a disproof of the statement in the *Annals* that there was distress. The planters were more concerned by the fact that the *milreis* return was the lowest in more than ten years. The *Statist*, the *Tea and Coffee Trade Journal*, the British Consul, Dr. Hermann Kurth, and Professor Augusto Ramos all testify that the planters were on the verge of financial ruin. The numerous petitions presented to the Camera dos Deputados add similar testimony.

Very truly yours,

WILLARD L. THORP

A RETRACTION

To the Editor of the Journal of the American Statistical Association:

Sir:

My present position on the question of the trend-seasonal normal in time series is not that expressed in my article in this JOURNAL for September, 1926. I have come to realize that the criterion there presented is not general as it should be but specific in respect of arbitrary periods of time, being based upon a careless misuse of the phrase "average activity for the year." I must apologize for going hastily into print prior to sufficiently careful reflection upon the matter printed. With the concept of "average activity for the year," I hope, properly in mind, I withdraw my objection to the usual method of combining measures of trend and seasonal.

Yours truly,

C. H. WHELDEN, JR.

AN ADDITION TO THE NEW COMPOSITE INDEX OF WAGES IN THE UNITED STATES

In the featured note on "A New Composite Index of Wages in the United States," by Carl Snyder, in the December, 1926, issue of this JOURNAL, page 466, the table of index numbers referred to at the end of the paper was omitted. It is given herewith:

FEDERAL RESERVE BANK COMPOSITE INDEX OF WAGES IN THE UNITED STATES

Based on:	1. Agricultural	Wages	(Weight)	5
	2. Railroad	"	"	10
	3. Teachers	Salaries	"	5
	4. Factory	Wages	"	40
	5. Building	"	"	15
	6. Clerical	"	"	15
	7. Unskilled labor	"	"	10

1913=100	1919	1920	1921	1922	1923	1924	1925	1926
January	165	192	207	185	196	209	212	217
February	163	193	203	183	195	209	211	215
March	164	201	203	186	200	211	214	218
April	165	203	200	183	201	209	211	218
May	167	208	198	186	206	209	212	218
June	170	211	193	187	208	208	214	218
July	173	212	189	187	208	208	213	219
August	177	213	190	190	207	209	213	219
September	181	213	188	192	207	211	214	221
October	181	214	188	194	210	210	216	221
November	186	212	186	195	208	210	215	220
December	190	210	187	196	209	212	217	222
Average	174	207	194	189	205	210	214	219

PROGRESS OF WORK IN THE CENSUS BUREAU

THE 1925 CENSUS OF AGRICULTURE: SUPPLEMENTAL TABLES

The series of state bulletins presenting statistics of agriculture by counties has been completed, and a bulletin has been published summarizing the data for the United States, by states. The state bulletins will be bound into three volumes covering the Northern, the Southern, and the Western States, respectively.

At the urgent request of the Secretary of Agriculture, the Bureau of the Budget approved an additional appropriation of $100,000 to enable the Bureau to tabulate and compile certain data which could not be provided for under the existing appropriations. The desired data included number of farms reporting each of the various crops and classes of livestock; classifications of acreage, values, farm expenditures, coöperative marketing, farm population, livestock and crops, by tenure and by size of farms; taxes; indebtedness, etc. Unfortunately the appropriation item was carried in the second deficiency bill, which Congress failed to pass. That being the case, it was decided to abandon the plan for a general report by subjects (corresponding to Volume V of the Fourteenth Census) in order to devote the remainder of the available funds to tabulating and compiling as much as possible of the data requested by the Department of Agriculture. Carrying out that program, four supplemental tables have been issued for each state, giving by counties the number of farms reporting each of the principal crops and each class of live stock; acreage of all farm land and of crop land harvested and value of land and buildings by size of farm; coöperative sales and purchases, by tenure; and farm population, by tenure. These supplemental tables will be included in the three bound volumes for the states.

BIENNIAL CENSUS OF MANUFACTURES: 1925

The series of preliminary reports of the 1925 biennial census of manufactures, published in mimeographed form, has now been completed, comprising 327 reports for individual industries, 16 reports each presenting figures by industries for some one of the 16 main groups of industries, and finally a summary report giving totals for all industries combined and for each of the 16 groups.

The preparation of final reports is well under way. Up to May 10 copy for 26 industry reports had been sent to the printer, and it is expected that the series will be completed not later than June 30. Copy for a summary report giving industry statistics for states and large cities will be sent to the printer about July 1, which is nearly four months in advance of the corresponding date for the census of 1923.

FINANCIAL STATISTICS OF STATES AND CITIES

The annual report on financial statistics of states, 1925, sent to the printer September 3, 1926, was available for distribution February 5, 1927. The annual report on financial statistics of cities having a population of over 30,000, 1925, which was sent to the printer October 26, 1926, was received in April,

1927. All tables and text for the 1926 report on financial statistics of states have been completed; and the report on the financial statistics of the 250 cities having over 30,000 population, 1926, will be completed in August, 1927.

THE PROBLEM OF ESTIMATING POPULATION

Estimates of population are necessary as a basis for computing per capita rates, particularly birth-rates and death-rates, and are very much in demand for the use of state and county officials, trade organizations, and business concerns. Therefore the Bureau of the Census can hardly avoid making estimates. But each successive year following a census widens the divergence between the unknown correct population and an estimate arrived at by projecting the increase of population shown by the last two censuses. Estimates of the total population of the United States, it is true, are no longer arrived at in that way, but are based on the records of births, deaths, and net immigration, so that the result can be accepted with a good degree of confidence notwithstanding the lapse of time since the last census. But this is a method which is not applicable to the individual state or to smaller political units, since we have no record of the migration to or from states and cities. In estimating the population of the states, however, the Bureau makes the assumption that the total increase of population for the United States since 1920 has been distributed among the states in the same proportions as the total increase between 1910 and 1920. This may be regarded as some improvement over the old method. But in estimating the population of cities and counties no better practicable method has been discovered than that of projecting the increase shown by the last two censuses; and since that method as applied seven years after a census yields very uncertain results, the Bureau has decided to discontinue making estimates for counties and for cities of less than 30,000 population from now until the census of 1930 is taken. This procedure follows the precedent established in 1917 when similar action was taken by the Bureau pending the census of 1920. The only remedy for this unsatisfactory situation appears to be a quinquennial census.

THE INTERNATIONAL LIST OF CAUSES OF DEATH

Dr. William H. Davis of the Bureau accompanied by Dr. Haven Emerson visited Geneva and Paris last March for the purpose of consulting with representatives from other countries regarding the revision of the International List of Causes of Death, which will take place in 1929. Dr. Davis reports that the outlook for a satisfactory decennial revision is very good.

J. A. H.

CAN DEPENDENCY AND RELIEF BE MEASURED?

The American Statistical Association met in joint session with the New York Chapter of the American Association of Social Workers on Friday evening, March 25, 1927, at the Town Hall Club, 123 West 43 Street, New York City. The number of persons present was 144.

Dr. Edmund E. Day, President of the American Statistical Association, introduced Mr. Bailey B. Burritt of New York, General Director of the Association for Improving the Condition of the Poor, who ably presided over the meeting. The general topic for discussion was, "Can Dependency and Relief be Measured?"

The first speaker of the evening was Dr. Maurice B. Hexter of Harvard University, also connected with the Federated Jewish Charities of Boston. He presented a most interesting series of charts representing changes in factors influencing dependency in Boston during the years 1875 to 1926. The charts recorded separately the seasonal movements, the cyclical movements, and the trend in each factor. The trends of income for most of the social agencies studied were sharply upward, the total income nearly trebling between 1912 and 1922. Dr. Hexter pointed out that, in a number of cases, there was distinct correlation between the cyclical movements of these social measures of dependency and the cyclical movements of business. His final goal was to secure an index measuring the fluctuations of dependency in Boston.

He stated that the amount of pauperism had for a long time been used as an index of dependency, but that, until Dr. Rubinow undertook the task, no one had thought of combining a large number of different items into a single index. Dr. Hexter presented a composite chart showing the cyclical movements in Boston dependency as indicated by such a combination of the various factors studied.

The second speaker of the evening was Miss Margaret H. Hogg, now of Smith College but formerly of the London School of Economics and Political Science. She presented a paper entitled, "Measurement of the Reduction of Poverty in England, between 1914 and 1924." Her study presented an analysis of data from inquiries originated by Professor Bowley with a view of comparing the condition of the working class in 1924 with its condition in 1913. The study covered several towns in England. For purposes of the investigation, poverty was defined as being that condition of the family existing when, after the deduction of national insurance and rent, the income was so low that only through the exercise of care and good management, could it be made to furnish a reasonable subsistence.

The findings of the inquiry were that the proportion of families in poverty, as defined above, declined from 11 per cent in 1913 to 6½ per cent in 1924. This indicates that, despite the large amount of unemployment, the English working class is distinctly better off than it was a decade ago. It must be remembered, however, that part of the present income is received from the unemployment dole and the fact should also be kept in mind that continual unsuccessful search for work tends to destroy the morale of those chronically unemployed.

Miss Hogg's study also indicated further that one of the chief reasons for the improvement of the condition of the British working classes is the decline in the number of dependent children per family, or, in other words, the increase in the relative proportion of wage earners which has occurred. As an immediate cause of poverty in England, low pay appears to be a disappearing factor, for the proportion of the families with a wage adequate to keep out of poverty a family possessing three children, sank from 4½ per cent in 1913 to ½ of 1 per

cent in 1924. In 1924, most of the poverty was immediately due to deaths in the family, old age, illness, and complete unemployment or only partial employment of the head of the family.

The third speaker of the meeting was Dr. Willford I. King of the National Bureau of Economic Research. He described briefly the preliminary investigation made by that organization for the Carnegie Corporation in an attempt to discover whether or not it is feasible to measure the changes that have occurred during the last 25 years in the extent of philanthropic giving. When plans were being laid for attacking this problem, it was decided that it is impossible to ascertain how much individuals have contributed to philanthropic and religious purposes, hence it was decided to do the next best thing and try to find out how much money has passed through the hands of philanthropic and religious organizations during each of the years in the last quarter century, the sources of the income of such organizations, and the nature of the expenditures which they have made.

New Haven was selected as the city in which the workability of this plan of attack could be tested. Better results were secured than were anticipated, the success of the undertaking being due in no small measure to the assistance given by Mr. John D. Dawson, Director of the Community Chest in New Haven. Out of 245 organizations discovered to be operating or to have operated in New Haven since 1900, some information was obtained from 234, and reasonably complete reports were secured from 149.

Dr. King explained that the charts presented covered only part of the organizations, the study still being incomplete. The graphs showed, however, that the total receipts of the organizations had been multiplied approximately by six during the 25 years between 1900 and 1925. Receipts of social agencies had, however, grown faster than those of religious organizations, the former having been multiplied by approximately ten and the latter by three.

Dr. King also called attention to the fact that social agencies show marked cyclical movements having a wave length of about four years. Figures for churches and synogogues show little trace of any cyclical oscillations.

This inquiry involved considerable expense as it required the time of two workers for nine months to make the canvass and this did not include the large amount of work necessary for the tabulation of the schedules.

The discussion was opened by Dr. I. M. Rubinow of the Jewish Welfare Society of Philadelphia. He pointed out that dependency is not the same thing as poverty. For example, a family may be moderately well-to-do and dependent, or very poor and non-dependent. It was his opinion that Professor Hexter would do well to construct an index of dependency covering a brief period of years and including the large number of criteria of dependency which are available for a short time but which cannot be secured for any considerable period. Dr. Rubinow felt that there is no doubt whatever that changes in dependency can be measured, for dependency indices fluctuate violently. The hope of forecasting social conditions by the use of index numbers of dependency is about on a par with the hope of forecasting business conditions by the aid of business barometers. Social agencies should be required to make annual reports to

proper authorities giving financial information of the type collected at New Haven by Dr. King and his assistants. Were such reports required, the expense of collecting the data would be reduced to a mere fraction of the present cost.

The discussion was continued by Mr. Linton B. Swift of the American Association for Organizing Family Social Work. He expressed some doubt as to the reliability of index numbers of dependency for he felt that it was very difficult indeed to define a dependent family and, hence, to find out how many dependent families there are. If a family receives a meal is it to be given the same weight in the index of dependency as a family that receives relief for a period of six months? Ought families receiving unemployment doles to be classed as dependents? Does free service at a state hospital constitute dependency? These are only a few of the questions that arise. Mr. Swift pointed out further that the case-loads of various societies differed not only because of changes in dependency, but also because of changes in the income of societies or the policies of relief. He thought that no existing index satisfactorily measures dependency, but he hoped that such an index might be constructed.

Dr. Emil Frankel, Statistician of the State Department of Welfare of Pennsylvania, expressed the view that it is extremely important to have available measures showing fluctuations in the amount of dependency relieved by social agencies, even though it is granted that these cases are not the sole cases of relief. It was his opinion that the more indicators that we collect representing relief per thousand population, the better possibility there is of understanding conditions of dependency in any given area.

Professor Robert E. Chaddock of Columbia University emphasized the need of having clear definitions and classifications before beginning to construct index numbers of dependency. The social worker should furnish the definitions and classifications. With these in hand, the statistician can construct the required index numbers.

Dr. Ralph G. Hurlin of the Russell Sage Foundation pointed out that it is possible to measure changes in dependency even though one has no adequate measures of the absolute volume of dependency. He disagreed with Mr. Swift's view that it was not possible to add apples, oranges, and plows in making up an index number, for he pointed out that, when fluctuations in the various items are reduced to relatives, it is perfectly legitimate to combine them to form an aggregate.

Dr. William A. Berridge of the Metropolitan Life Insurance Company endorsed the view expressed by Dr. Hurlin. He pointed out that it is just as reasonable to utilize different social factors to construct an index of dependency, as to put together production figures for pig iron, shoes, and numerous other articles to form an index number of production.

The discussion was closed by Dr. F. Stuart Chapin of the University of Minnesota. He pointed out that indices are often very useful, even if the inner nature of the data composing the indices is not clearly understood. For instance, even though no one has yet discovered the smallpox germ, the United States Public Health Service finds it extremely useful to know the weekly fluctuations in the number of cases of smallpox, for such information aids greatly in the control of that disease. The meeting adjourned.

FORECASTING THE VOLUME OF ORDERS OR SALES

A dinner meeting of the American Statistical Association was held at the Aldine Club, 200 Fifth Avenue, New York City, on the evening of February 25, 1927. Ninety-four people were present.

Mr. F. Leslie Hayford, Statistician for the General Motors Corporation of New York, presided. He opened with a quotation taken from William Trufant Foster's prefatory note in *The Problem of Business Forecasting:*

> When a man enters business, he enters the forecasting profession. He may forecast badly or well, but forecast he must. He may scorn business forecasters but he cannot help being one. He may shun statistics, but he cannot help using them. Since business is essentially risk-taking with the expectation of profits, every enterpriser must run risks; and, as a risk-taker, he is necessarily a business forecaster.

Mr. Hayford pointed out further that the basic type of forecast which a business man must make is a forecast of orders or sales, for production and operating schedules are all based upon anticipated orders.

The first speaker of the evening was Mr. Paul T. Cherington of the J. Walter Thompson Company, New York City. Mr. Cherington expressed the view that one of the chief handicaps of business is the presence of ignorant and obstinate executives who are either unable to comprehend a statistical analysis of their business, or are unwilling to heed the obvious teachings of the statistical findings presented by the statisticians in their employ. He also called attention to the fact that statisticians sometimes fail to relate the statistical problem adequately to the personal factors in the business.

He next proceeded to describe the results of some of his attempts to apply to the records of five business concerns "S" shaped growth curves of the type advocated by Mr. Ray B. Prescott. He found that, after the installation of a new sales policy, there is a distinct tendency for business to follow this type of curve. At first the new policy must overcome the inertia of the consumers and the curve rises slowly; then there is a period when it produces good results and the curve ascends rapidly; later the general type of appeal loses its novelty and sales slow up, the result being that the curve flattens out. Sales finally die down so much that it pays to adopt a new plan of advertising or sales stimulation. When this is done the curve is likely to be repeated again.

Some of his findings were, however, less cheering to curve disciples, for he found that, in two cases out of the five studied, the experience of business concerns did not fit in well with these general principles. Furthermore, the length of time taken to pass through a given phase differed in the three different concerns in which all the phases followed each other in the expected order. This variation in the length of phase, of course, increases the difficulty of forecasting.

Mr. Cherington desired to emphasize one point which he said was all too commonly overlooked, namely, that the business man is not primarily interested in ascertaining the future course of industry in general. He hopes and expects that his enterprise will secure a constantly larger proportion of the business in

the field, and what he is most anxious to have, is a forecast of orders or sales for his own undertaking.

The second speaker of the evening was Mr. Walter P. Hedden, Chief Analyst of the Port of New York Authority. For some time he has been engaged in the study of the marketing of perishable foodstuffs in New York City, and, in the course of this investigation, many interesting facts have been brought to light. Among those which he mentioned are the following: Retail spoilage in the case of fresh fruits and vegetables amounts to approximately 14 per cent; of the milk sent out for delivery in northern New Jersey, 3 to 4 per cent is returned unsold; the percentage of cream unsold sometimes runs as high as 10 per cent.

The changes in prices of perishable products due to variations of the supply are simply tremendous. For example, the price of lettuce has been known to decline from 18 cents a head to 2 cents a head within a period of three weeks. That at times the market is glutted with an enormous over-supply of vegetables and fruits is illustrated by the fact that there have been on the tracks in New York as many as 425,000 watermelons at one time. The receipts of lettuce have run as high as 1,500,000 heads in a single day, and, on another day, 15,708,000 peaches, or more than two for every person in New York City, appeared on the local market. In such instances, the total sales-value of certain grades of these products falls too low to pay the freight. At such times terminal congestion may become so great that the railroads declare an embargo on deliveries of these perishable goods. Merely adding trackage facilities will not relieve the shipper from losses. A better regulation of shipments in accord with careful studies of what the market can absorb is also needed.

The aim of the study of market demand for perishable foodstuffs is to find the best way of controlling marketing policy rather than production quotas. The volume of agricultural production and the time of maturity of crops is so largely influenced by weather that control is difficult. Moreover, with few exceptions, there is no organization in a position to control the scattered independent producing units.

What the shipper needs to know on any specific date is what grades of vegetables or fruits it will pay him to ship to market. Without this knowledge, he will, at the height of the season, send in large quantities of produce which will not sell for enough to pay shipping costs. In fact, at such times, only the best grades give any net return to the grower. Monthly sales forecasts are of no particular value to the producer of perishables. What he needs is information concerning the probable action of the markets during the next day or week.

Mr. Hedden called attention to the fact that economists have frequently been guilty of making an erroneous assumption when they have taken it for granted that a correct demand curve can be constructed from records of production and selling prices. In the case of perishable foodstuffs, this is far from being true, for, since a large proportion of the product never goes to market, there is no necessary relationship between the volume of production and the volume of sales. The only way, then, to derive a demand curve having any merit is to base it upon the volume of the actual sales to the consumer in comparison with the prices prevailing when the sales are made.

Investigation has revealed the fact that the demand for many kinds of foodstuffs is very inelastic. This is true in the case of milk. It is, however, incorrect to suppose that the shape of the demand curve remains practically unchanged over long periods of time. It is, for instance, materially affected by changes in the weather. Furthermore, the curve differs greatly on different days of the week—for example, the sales of cream on Thanksgiving are seven times as great as the sales on any other day in the year. It is hard to over-rate the influence on sales and market prices of differences in temperature, of the day of the week, of holidays, etc.

The third speaker of the evening was Mr. Ray B. Prescott. He pointed out that not only industries as a whole, but also individual business enterprises normally pass through four stages. First, the period of experimentation when they are overcoming inertia and growth is slow. Second, the period of exploitation when the inertia of the customers has been overcome and growth is rapid. Third, the period of retardation as competition springs up or the needs of the market become saturated. Fourth, the period of stability when, in the main, sales merely take care of replacements. In most cases the growth of an industry or of an enterprise can be fitted by an "S" shaped curve of the Gompertz or Pearl-Reed variety. These curves typify the growth of plants and lower animals, but they seem to be equally applicable to the development of human activities. The laws of growth have existed from time immemorial; slowly they have been discovered by man.

In any given firm or industry it takes time to discover what the characteristics of its growth are. It is almost useless to attempt to fit a curve to the records of an industry or enterprise that are available for only four or five years. Furthermore, one cannot fit an "S" shaped curve to the data until it has reached the point of inflection and has begun to turn towards the horizontal.

The last of the regular speakers was Mr. Victor S. Von Szeliski, a New York statistician. He emphasized the necessity of differentiating between records of inventories and records representing the current activities in an industry. He illustrated his point by showing the difference between a record of checking-account balances on various dates and a corresponding record of deposits and withdrawals of money from the bank. The bank balance corresponds mathematically to a stock of goods on hand, while deposits and withdrawals resemble curves relating to production or orders. Different rules must be used in interpreting curves regarding the stocks on hand, from those used in the analysis of curves of the other type. One should always keep in mind the fact that the curves representing stocks tend to vary because of additions or subtractions of definite amounts and not in fixed ratio to former amounts.

Mr. Von Szeliski discussed various methods of interpreting data on inventories or stocks on hand. One method is to express these inventories as the day's supply for the market. When expressed in this fashion, one should remember that the sales on various dates differ greatly in volume, so that the same stock of gasoline, for example, may constitute ten days' supply at one season of the year, and twenty days' supply at another.

The speaker also called attention to the difficulty of eliminating the seasonal

with any degree of accuracy. He felt that most of the irregularities of the typical curve remaining after the seasonal has supposedly been taken out are due to improper elimination of the seasonal. Another point emphasized was that the best way to forecast any given series is by a study of the past record of the series itself. The speaker believes that such a record nearly always contains information which, if properly extracted, will tell what is going to happen.

One of the discussants was Edward T. Frankel, Statistician of the National Industrial Conference Board. He felt that Mr. Von Szeliski was in error in assuming that a curve forecasts itself. Mr. Robert W. Burgess of the Western Electric Company agreed with the view of Mr. Frankel and stated that, in his opinion, it is impossible to predict the future movements of a variable with any degree of accuracy unless one can find one or more other variables the movements of which run ahead of the one which it is desired to forecast. He has found multiple correlation extremely helpful in estimating the influence of the various factors and has, in this manner, been able to prophesy with some degree of accuracy the actual course of orders.

Mr. Knapp called attention to the fact that, for use in analyzing figures on stocks of commodities, the rate of change is often much more important than the actual size of the stocks. This is especially true in the correlation or changes in inventories with the changes in price. He stated that, in the case of gasoline, it is as a rule, possible to forecast the volume of business for the next three months within 3 per cent of the actual figures.

Mr. William A. Berridge stated that he felt that the audience would be interested to know that the forecast of the total production of passenger automobiles made by Mr. Prescott in 1921 had been fulfilled with a strikingly small error.

The Chairman, in closing, gave it as his opinion that short-time forecasting is much more important to the average business man than telling what is going to happen in the long run; for production schedules are based upon the volume of orders anticipated for the next two to four months. His experience has shown that each company must develop its own forecasting system, but that a knowledge of the general principles of forecasting is essential for building up the system for the individual concern. He commended highly an article by Donald Smith in the last issue of the *Harvard Business Review*.

The meeting adjourned.

MISCELLANEOUS NOTES

United States Bureau of Labor Statistics.—Since the last quarterly report, studies have been started on the following subjects: Productivity of labor in the iron and steel industry; productivity of labor and labor costs in loading and unloading vessels; and state and municipal retirement plans. The investigation of Mexican colonies in the United States has been partly completed and a preliminary report is being prepared. Other studies now under way are: Apprenticeship in the building trades; wages and hours of labor in the textile, boot and shoe, and hosiery and underwear industries.

Columbus, Ohio, Statistical Dinner.—The Columbus Chapter of the American Statistical Association held its fourth and closing session Tuesday evening, March 29, at the Faculty Club, Ohio State University, with an attendance of twenty-eight. After the dinner, a symposium on the general topic "Objectives in the Field of Statistics" was held. The speakers and topics were as follows:

(a) "Why a Discussion of Statistical Objectives?" by Dr. Harry S. Will, President, Columbus Chapter of the American Statistical Association.
(b) "Pioneering in Applied Statistics," by Mr. Frederick C. Croxton, Secretary, Columbus Community Fund.
(c) "Utilizing Statistics in the Field of Education," by Dr. Sidney L. Pressey, Professor of Psychology, Ohio State University.
(d) "What Business Asks of the Statistician," by Dr. Spurgeon Bell, Director Bureau of Business Research, Ohio State University.
(e) "Statistical Technique in the Service of Agriculture," by Dr. Carl J. West, Federal Crops Statistician, Columbus.

The discussion which followed the addresses gradually centered about the question as to what sort of training should be given prospective statisticians in order that they might be fully prepared to handle the varied problems that are being referred to the statistical profession at the present time.

A motion was carried for the appointment of a committee consisting of experienced theoretical and practical statisticians to consider further the questions raised in the discussion and to draw up a set of resolutions stating their conclusions as to what technical and supplementary training should be given statisticians in the several recognized fields. The committee will be expected to report some time next year.

The officers for the ensuing year are: President: Dr. Harry S. Will; Secretary-Treasurer: Dr. Herbert A. Toops; Counsellors: Dr. Spurgeon Bell, Mr. Frederick C. Croxton, Dr. Carl J. West.

Welfare Council of New York.—Statistical measurements have been used in a study of employment agencies for the handicapped in New York to discover the possibility of better coördination of work and better service to handicapped wage-earners. The results have just been published by the Welfare Council of New York in a pamphlet entitled *Securing Employment for the Handicapped*, by Mary LaDame of the Department of Industrial Studies of the Russell Sage Foundation, who made the investigation at the request and under the auspices of a committee representing several of the agencies. The statistical tables indicate the number and type of applicants applying to twelve of these agencies in which uniform records were kept simultaneously during a period of four weeks. Particularly significant are the data on the handicaps of applicants; the service rendered to them, including the jobs in which they were placed; and the weekly rate of wages received. As a result of the study four of the agencies under the auspices of the Welfare Council are planning to coördinate their efforts in a joint bureau. In view of the wide interests in the problem and the provision by federal and state governments for a program of rehabilitation of the handicapped in which a large majority of states are coöperating, these facts on securing employment, which is one of the most vital phases of rehabilitation, are timely and should stimulate similar inquiry in other communities.

OBITUARY NOTES

Keppele Hall.—By the death of Colonel Keppele Hall on April 25, 1926, the Association loses a beloved member.

Early associated with Frederick Winslow Taylor, he brought to the movement of Scientific Management a rare combination of engineering temperament and human sympathy. He was an international authority on management technique, as well as a frequent and trusted arbitrator in labor disputes.

On the General Staff of the Army, Colonel Hall worked out the unified method of Ordnance production and distribution used during the war. As a consultant on management he was associated with the firm of Cooke, Hathaway and Williams, and consequently came in contact with many of the largest industrial enterprises of the country.

During the last few years of his life, Colonel Hall devoted himself to the application of statistics to business technique in the fields of material and production control, personnel, costs, and budgets. Although he was an infrequent contributor to technical journals, his wide association with business executives, his thorough knowledge of the science of management, and his engaging personality made his influence on the technique of management and the development of industrial statistics both extensive and profound.

Colonel Hall joined the American Statistical Association in 1924.

Charles H. Angell.—Charles H. Angell, Vice-President of the Massachusetts Mutual Life Insurance Company, and a prominent member of the Springfield School Board, died at Springfield, Massachusetts, on January 17, 1927.

Mr. Angell was graduated from Princeton University in 1893 with the degree of Bachelor of Arts. He at once entered the employ of David Parkes Fackler in New York. After five years' work there he joined the actuarial department of the Massachusetts Mutual in Springfield, where he was advanced to Actuary of the company. From that position he became a member of the Board of Directors and finally Vice-President. Mr. Angell has been a member of the American Statistical Association since 1924.

Judge Albert D. Bosson.—Albert D. Bosson, a member of the American Statistical Association since 1904, was born in Chelsea, Massachusetts, November 8, 1853, the son of George Chapman and Jennie Hood Bosson. He was educated in the Chelsea public schools, Phillips Exeter Academy, and graduated from Brown with the class of 1875. After studying law he was admitted to the Bar and practiced in Boston for many years. He was Mayor of Chelsea for one year and was made Associate Justice and later Justice of the Chelsea District Court, a position which he held for over thirty-five years until the time of his death. He was one of the founders and the first President of the County Savings Bank of Chelsea, in the presidency of which he continued to the time of his death. He also served as President of the Winnisimmet National Bank of Chelsea. He was very much interested in travel and history, and wrote several short articles on subjects connected with the history of Chelsea. Mr. Bosson died on Easter Day, 1926.

PERSONAL NOTES

Dr. William Leonard Crum, now Assistant Professor of Economics at Harvard University and Director of Statistical Laboratory of the Harvard Committee on Economic Research, has resigned his position at Harvard and has accepted appointment as Professor of Statistics in the Graduate School of Business at Stanford University, California, beginning September 1, 1927.

Reuben D. Cahn retired by resignation from the position of Chief of the Bureau of Industrial Accident and Labor Research of the Illinois Department of Labor, on

January 15, 1927, after a service with the Department of five and one-half years. Mr. R. L. Forney has been appointed Acting Chief of the Bureau.

Dr. Raymond Pearl, Director of the Institute for Biological Research of the Johns Hopkins University, will give a series of lectures on "Experimental Vital Statistics" at the University of London early in June. The lectures will subsequently be issued as a book.

Professor Sven D. Wicksell, who holds the Chair of Mathematical Statistics at the University of Lund, Sweden, has been spending the second semester as special lecturer at the University of Michigan. There has been very gratifying interest shown in his two courses, one on Advanced Theory of Frequency Distributions and Correlation, the other on Stellar Statistics, particularly considering their advanced nature. At the April 5th meeting of the University Mathematical Club he presented a paper on some of his researches in mathematical statistics.

COMMITTEE APPOINTMENTS, 1927

Social Science Research Council — Term Expires

H. L. Rietz ..... December 31, 1927
Edmund E. Day ..... December 31, 1928
Wesley C. Mitchell ..... December 31, 1929

Advisory Committee on the Census

S. L. Andrew ..... December 31, 1927
W. S. Rossiter ..... December 31, 1928
Robert E. Chaddock ..... December 31, 1929

Joint Committee on Standards for Graphics

Karl Karsten, Chairman
Irving Fisher
Arthur H. Richardson
Howard Green

Joint Committee on the Encyclopedia of the Social Sciences

Mary Van Kleeck, Chairman
R. H. Coats
Wesley C. Mitchell

Committee on Institutional Statistics

Horatio M. Pollock, Chairman
Kate H. Claghorn
Edith M. Furbush
Joseph A. Hill

Committee on Governmental Labor Statistics

Mary Van Kleeck, Chairman
A. J. Altmeyer
Charles E. Baldwin
Joseph A. Becker
William A. Berridge
Louis Bloch
W. Randolph Burgess
R. D. Cahn
R. H. Coats
Frederick E. Croxton
Bryce Stewart
J. F. Dewhurst
Leonard W. Hatch
Ralph G. Hurlin
Richard Lansburgh
Don G. Lescohier
Eugene B. Patton
Roswell F. Phelps
Woodlief Thomas
Fred G. Tryon
Howard H. Ward
Paul H. Douglas

Committee on Finance

Edmund E. Lincoln, Chairman
Frederick R. Macaulay
Leonard P. Ayres

Membership Committee

Edwin W. Kopf, Chairman
W. Randolph Burgess
A. H. Mowbray
Jessamine S. Whitney

Committee on Advertising

Paul T. Cherington, Chairman
L. D. H. Weld
S. O. Martin
Philip J. Warner
Warren F. Hickernell

Committee on Social Statistics

Ralph G. Hurlin, Chairman
George Bedinger
William A. Berridge
F. Stuart Chapin
Neva Deardorff
Emma A. Winslow
Emil Frankel
Hornell Hart
Philip Klein
Linton B. Swift

Committee on Fellows

	Term Expires
Joseph A. Hill	December 31, 1927
Raymond Pearl	December 31, 1928
Louis I. Dublin	December 31, 1929
Robert E. Chaddock	December 31, 1930
Horace Secrist	December 31, 1931

Committee on Nominations

Robert E. Chaddock, Chairman
Louis I. Dublin
Leonard P. Ayres

MEMBERS ADDED SINCE MARCH, 1927

Abbott, Mrs. William S., 965 Madison Avenue, New York, N. Y.

Anderson, Robert Emery, Jr., Merchandising Research Department, Pacific Mills, Boston, Mass.

Austin, Edward R., Investment Analysis, Bodell and Co., 32 Custom House Street, Providence, R. I.

Beal, Arthur F., U. S. Public Health Service, Washington, D. C.

Bishop, Granville C., Business Statistics, The Pacific Telephone and Telegraph Co., San Francisco, Calif.

Blackburn, William J., Jr., Instructor, College of Commerce, Ohio State University, Columbus, Ohio.

Bodfish, H. Morton, Bureau of Business Research of Ohio State University, Columbus, Ohio.

Booton, John G., Major, Ordinance Department, 95 Mason Terrace, Brookline, Mass.

Brandt, A. E., Instructor, Iowa State College, Ames, Iowa.

Burdick, W. A., Statistician, Chamber of Commerce, Rochester, N. Y.

Burnham, Frederick E., Economic Forecasts, Blakemore Analytical Reports, Inc., New York, N. Y.

Bursk, J. Parker, Instructor in Statistics, Wharton School, University of Pennsylvania, Philadelphia, Pa.

Card, Dana G., Statistical Work at Kentucky Agricultural Experiment Station, Lexington, Ky.

Carson, J. V., Auditing and Cost Accounting, Title Insurance and Trust Co., Los Angeles, Calif.

Carter, Burnham, Publicity, Ivy Lee, 111 Broadway, New York, N. Y.
Chambers, Benjamin P., Statistician, 229 Seventy-Third Street, Brooklyn, N. Y.
Clapp, Elizabeth V. W., Social Research, Institute of Social and Religious Research, 370 Seventh Avenue, New York, N. Y.
Dewey, Ralph L., Instructor in Economics, Ohio State University, Columbus, Ohio.
Diamond, Jay G., Agricultural Statistician for Montana, U. S. Bureau Agricultural Economics, Division of Crop and Livestock Estimates, Helena, Mont.
Dickerson, Dr. William E., Instructor, University of Chicago, Chicago, Ill.
Doerschuk, Beatrice, Statistics of Social Work, Welfare Council of New York City, New York, N. Y.
Dolbeare, Carl B., Babson Statistical Organization, Wellesley Hills, Mass.
Doolittle, J. Edison, Statistical Department, Southern New England Telephone Co., New Haven, Conn.
Dorr, Bryan R., Student, Harvard Graduate School of Business, Cambridge, Mass.
Dowe, Mabel E., Statistician, Playground and Recreation Association of America, 315 Fourth Avenue, New York, N. Y.
Drake, Francis R., Gilfillan Bros., Inc., 25 Wilbur Avenue, Long Island City, N. Y.
Dunnigan, William, Farmer, Chochilla, Calif.
Duryea, Rhea E., Federal Reserve Bank, Philadelphia, Pa.
Edgerly, Lloyd I., Manager, Commercial Research Department, Edison Lamp Works, Harrison, N. J.
Fenton, John M., Marketing Specialist, Bureau of Markets, New Jersey Department of Agriculture, Trenton, N. J.
Ferger, Wirth F., Assisting in the University of Wisconsin, Department of Economics, Madison, Wis.
Folse, J. A., Research in Hydrology, Carnegie Institution of Washington, Washington, D. C.
Ford, Donald R., 140 West Street, New York, N. Y.
Gaillard, D. P., Investment Statistics and Analyses, 839 Seventeenth Street, N. W., Washington, D. C.
Garver, Frederic B., Professor of Economics, University of Minnesota, Minneapolis, Minn.
Gaubis, Anthony, Junior Economic Statistician, American Telephone and Telegraph Co., New York, N. Y.
Ginsberg, Helen, Investments and Retail Merchandising, R. H. Macy and Co., New York, N. Y.
Gleason, Mary H., Federal Reserve Bank, Chicago, Ill.
Greissman, Jacob, Advertising and Market Statistics, New York Evening Journal, New York, N. Y.
Grindal, Herbert W., Banking and Investments, Lehman Bros., 16 William Street, New York, N. Y.
Hall, Mary Bradt, Economic Statistics Division, American Telephone and Telegraph Co., 195 Broadway, New York, N. Y.
Halpern, Max, Statistician, Gruntal, Lilienthal and Co., 60 Broadway, New York, N. Y.
Hammerich, Henry O., The Pacific Telephone and Telegraph Co., 140 New Montgomery Street, San Francisco, Calif.
Hansen, Sophie, Analyzing Construction Statistics, McGraw-Hill Publishing Co., Tenth Avenue at 36 Street, New York, N. Y.
Herwitz, Harry K., Vice-President, Amalgamated Bank of New York, New York, N.Y.

Hewitt, R. I., Assistant Treasurer and in charge of all statistical work, General Motors Acceptance Corporation, 250 West 57 Street, New York, N. Y.

Ho, Dr. Franklin L., Professor of Finance and Statistics, Nankai University, Tientsin, China.

Hobbs, Ernest S., Interstate Commerce Commission, Washington, D. C.

Hoffman, Aaron A., Western Electric Co., 195 Broadway, New York, N. Y.

Honigman, J. Leo, Forecasting, 46 West Street, New York, N. Y.

Howard, Philip Williams, School of Hygiene and Public Health of the Johns Hopkins University, Baltimore, Md.

Horst, E. Clemens, President, E. Clemens Horst Co., Exporters, San Francisco, Calif.

Huckins, Harold A., Hood Rubber Co., Watertown, Mass.

Huff, Herbert M., Student, Harvard Graduate School of Business Administration, Cambridge, Mass.

Hull, Dr. Clark L., Director of the National Correlation Bureau, University of Wisconsin, Madison, Wis.

Jamba, Ann, Student, London School of Economics, London, England.

Johnson, Thor, Equitable Trust Co., 37 Wall Street, New York, N. Y.

Jones, Howard L., C.P.A., Illinois Bell Telephone Co., 309 West Washington Street, Chicago, Ill.

Kavanagh, John A., Brookmire Economic Service, 570 Seventh Avenue, New York, N. Y.

Kazakevich, V. D., Guaranty Trust Co. of New York, New York, N. Y.

Keller, Marvel, National Bureau of Economic Research, University of Wisconsin, Madison, Wis.

Keuker, Frans K. F., Student, Columbia University, New York, N. Y.

Kozelka, Richard L., Instructor, School of Business, University of Minnesota, Minneapolis, Minn.

Larus, John R., Phoenix Mutual Life Insurance Co., Hartford, Conn.

Link, Leo R., Statistical Department, Pacific Telephone and Telegraph Co., San Francisco, Calif.

Lin, Peter W., Student, Columbia University, New York, N. Y.

Longstaff, H. P., Instructor in Psychology, Ohio State University, Athens, Ohio.

Low, Josiah O., Investment Managers Co., 22 William Street, New York, N. Y.

Lucia, Mrs. Eschscholtzia L., Instructor in Vital Statistics, Department of Hygiene, University of California, Berkeley, Calif.

McCarteney, Summerfield, Statistical Department, Pacific Telephone and Telegraph Co., Seattle, Wash.

McCormack, James E., Student, Stanford University, Calif.

McMorris, Leon G., Statistician, H. N. Whiting and Sons, 49 Wall Street, New York, N. Y.

McNeil, Earl R., Bureau of Markets, State Department of Agriculture and Markets, Albany, N. Y.

Martz, C. E., Teaching Social Science, Cleveland School of Education, Cleveland, Ohio.

Miskelly, Lawrence W., Stocks and Bonds, 25 Melinda Street, Toronto, Ontario, Can.

Mitchell, George W., United Typothetae of America, Chicago, Ill.

Mitchell, Waldo F., Professor of Business Administration, Lawrence College, Appleton, Wis.

Moore, Thomas T., Student, Columbia University, New York, N. Y.

Nayhouse, Lucy, Retail Bureau, University of Pittsburgh, Pittsburgh, Pa.

Nelson, Edward G., Bureau of Statistics, Merchants National Trust and Savings Bank, Los Angeles, Calif.

Nerlove, S., Instructor of Risk and Risk-Bearing, University of Chicago, Chicago, Ill.

Newcomer, William L., The Brookmire Economic Service, Inc., 570 Seventh Avenue, New York, N. Y.

Newman, Mrs. Carey K., General Economic Research, American Telephone and Telegraph Co., 195 Broadway, New York, N. Y.

O'Neill, Grover, President, Joint Investors, Inc., New York, N. Y.

Onthank, A. Heath, Director Research and Merchandising, George Harrison Phelps, Inc., Advertising, Detroit, Mich.

Palyi, Dr. Melchior, University of Chicago, Chicago, Ill.

Pearson, John W., Analyses of Investment Values, First Investment Co., 3 Pleasant Street, Concord, N. H.

Pellett, Miriam D., National Bureau of Economic Research, New York, N. Y.

Peters, James C., Instructor, Central High School, Superior, Wis.

Phipps, Irving Rogers, Economic Statistics Division, American Telephone and Telegraph Co., 195 Broadway, New York, N. Y.

Pierce, Stephen S., Analysis of Sales and Operating Statistics, United Fruit Co., Boston, Mass.

Potter, Marguerite, East Harlem Health Center, 345 East 116 Street, New York, N. Y.

Rauchenstein, Dr. Emil, University of California Agricultural Experiment Station, Berkeley, Calif.

Regan, Arthur C., Minneapolis Trust Co., Minneapolis, Minn.

Rice, Dr. J. Nelson, Instructor, Catholic University of America, Washington, D. C.

Rollinson, Ethel A., Statistical work at Columbia University, New York, N. Y.

Rowell, Dorothy C., Student, Teachers College, Columbia University, New York, N. Y.

Ruch, Dr. Giles Murrel, Professor of Education, University of California, Berkeley, Calif.

Ryan, Paul A., Economic Statistics Division, American Telephone and Telegraph Co., 195 Broadway, New York, N. Y.

Schabacker, Richard W., Forbes Magazine, 120 Fifth Avenue, New York, N. Y.

Seem, Guy W., American Bond and Mortgage Co., 345 Madison Avenue, New York, N. Y.

Shipman, George J., Forecasting, American Telephone and Telegraph Co., 15 Dey Street, New York, N. Y.

Skalar, Charles, 1 Adrian Avenue, New York, N. Y.

Spiegelman, Mortimer, Statistical Bureau, Metropolitan Life Insurance Co., 1 Madison Avenue, New York, N. Y.

Starr, Myrtle M., Graduate Student, University of Wisconsin, Madison, Wis.

Stecher, F. H., Jr., Surveying Cleveland Market, The Cleveland Plain Dealer, Cleveland, Ohio.

Steiner, Paul M., Public Accounting, Wolf and Co., 33 West 42 Street, New York, N. Y.

Stephens, Philip H., Assistant in Marketing, Cornell University, Ithaca, N. Y.

Stewart, Dr. Bryce M., Industrial Relations Counselors, Inc., 165 Broadway, New York, N. Y.

Sunley, William T., Business Statistics, International Accountants Society, Inc., Chicago, Ill.

Sutcliffe, William G., Instructor, Simmons College, Boston, Mass.
Suzuki, Gengo, Graduate Student, University of Wisconsin, Madison, Wis.
Tutt, Ernest L., U. S. Bureau Foreign and Domestic Commerce, Houston, Texas.
Van Dyke, Charlena, Association for Improving Conditions of the Poor, 105 East 22 Street, New York, N. Y.
Vezin, Charles, Jr., Investment Counsellor, Scudder, Stevens, and Clark, 111 Broadway, New York, N. Y.
Wheeler, Oliver P., Manager, Division of Analysis and Research, Federal Reserve Bank, San Francisco, Calif.
Wissler, Willis, Industrial Research, Bureau of Business Research, Ohio State University, Columbus, Ohio.
Wolinsky, Moses, Teacher and Accountant, 1839 Andrews Avenue, New York, N. Y.
Working, E. J., Instructor, Division of Agricultural Economics, University Farm, St. Paul, Minn.
Yoney, Joseph G., First National Bank and Trust Co., Monessen, Pa.
Young, Harold H., Chief, Statistical Department, Bodell and Co., Providence, R. I.
Zimmerman, Maurice, Standard Statistics Co., 47 West Street, New York, N. Y.

REVIEWS

Tax-exempt Securities and the Surtax, by Charles O. Hardy. New York: The Macmillan Company. 1926. xx, 213 pp.

This is another of the monographs issued by the Institute of Economics, under the direction of Mr. Harold G. Moulton. As stated in the preface, popular interest in the subject is not so great now as when the study was initiated, but it is well that this investigation was completed, as it is concerned with matters of both theoretical and practical importance. It is especially useful in showing how many arguments can be and have been used by politicians and publicists without any statistical basis, and with very little theoretical justification.

As is well known, the system of taxing incomes at highly progressive rates, and at the same time permitting the owners of property to avoid paying these rates by issuing tax-exempt securities, has been the cause of violent controversy in the past seven or eight years. Dr. Hardy attempts to show that this system has not really been as absurd and harmful as it seemed, because the amount of tax-exempt securities actually used for the purpose of evading income taxes was less than generally supposed, and that the loss of revenue to the Federal Government caused by their exemption was less than the benefits received by the issuing authorities. This loss in 1923 is estimated at $71,000,000 on state and municipal bonds; $12,000,000 on Federal Farm loan bonds; and something over $16,000,000 on wholly exempt Federal Bonds. On the other hand, the value of the exemption to the states and municipalities in 1923 is reckoned at $75,000,000; the subsidy to farmers at $10,000,000; and the saving in interest to the Federal Government on the wholly exempt issues is believed at least to have equalled the loss of revenue. Neither the loss nor the saving on the partly exempt issues is estimated, but Dr. Hardy believes that, "the experiment of partial tax-exemption has been a colossal failure," apparently not because there was some exemption but because there was not enough.

The author disclaims any attempt at forecasting, and offers no opinion as to the expediency of exempting new issues of Federal bonds from Federal taxation, but merely describes the effects of exemption up to date, with a very ingenious and valuable collection of figures showing the amounts of various kinds of exempt securities outstanding and the probable distribution of their ownership. No one can study these figures without being impressed with the proof they afford that most investors have been very slow to change their investments in response to changing tax laws, whether from ignorance or inertia. Nevertheless it may be that this slowness was due largely to their belief that the law would soon be changed, and as there is ample evidence that the members of the higher income groups were gradually responding to the situation, it is reasonable to believe that if the high surtax rates in force from 1917 to 1923 had continued the response would have been more and more complete.

In general, the argument of the book is destructive rather than constructive, which is not necessarily a bad thing. It seems to the reviewer, however, that the destructive criticism has been carried too far in some respects. For example, in his effort to show that tax-exemption has not had the harmful effects on private financing that have been alleged, Dr. Hardy goes so far as to say that "Pensions and bonuses absorb no capital," being "merely a transfer of purchasing power from one set of individuals in the community to another set." It may be true, as he states on the following page, that such a transfer of funds that would otherwise have been expended for investment, to individuals who will expend them for consumption goods, does not directly decrease the wealth of the country and might under some circumstances be a stimulus to industry; but it certainly slows up the growth of capital and might result in a decrease of the nation's capital if the amount of bonus issues were large and the amount of normal capital accumulation small.

It would be possible for an economist to take Dr. Hardy's figures in so far as they purport to be exact, and by a difference in emphasis and in judgment concerning the admittedly inexact items, to arrive at conclusions that would differ considerably from his. In the reviewer's opinion the chief defect in the statistical part of the book is the inadequate treatment of the $18,000,000,000 of securities only partly exempt from surtax. Inasmuch as up to July 2, 1923, it was possible for an individual to hold $160,000 of these wholly free of tax, and as they yielded more than the wholly exempt issues, it is reasonable to suppose that they were held to a large extent by wealthy persons for the purpose of reducing their taxes, thereby reducing the amount of other issues required for that purpose. The figures in Dr. Hardy's book relate mainly to June 30, 1923, at which time the amount of such securities actually exempt must have been considerably greater than in 1924, since in 1924 the exemption was only $55,000 for each individual, and since moreover the total outstanding had declined. It is interesting to note that in 1924 the average holding of tax-payers with incomes over $300,000 was about $63,000, *i. e.*, slightly more than the exemption. The published figures for 1924 show that over $73,000,000 of interest on such bonds was received by individuals with incomes over $5,000 and indicate rather plainly that about $42,000,000 was actually exempted. The principal thus exempted must have been over $1,000,000,000. Assuming that the proportion of partly exempt securities outstanding that was held by individuals was the same as the proportion estimated by Dr. Hardy for wholly exempt securities, we get over $4,500,000,000 exempt from normal tax, and, as just intimated, at least $1,000,000,000 of this was exempt also from surtax. At the very lowest estimate, the taxes lost were over $20,000,000. In the reviewer's opinion the amount was much greater.

There are indications in the Estate tax returns that 1924 was the year of maximum employment of tax exempt securities to avoid the Federal Income tax. If this is true, it tends to strengthen the argument that low surtaxes mean less evasion and gives more justification to the Secretary of the Treasury's tax-reduction program than Dr. Hardy is willing to admit. However, he is perfectly justified in arguing that there are many other ways of avoiding high surtaxes

than by investing in tax exempts, and also that there are stronger arguments against high surtaxes than the possibility of encouraging investment in tax exempts. There are no doubt many economists and political scientists who would disagree violently with Dr. Hardy as to the advisability of a Federal subsidy by means of tax exemption to even such worthy objects as schools and highways. Nevertheless the book must be recognized as a valuable contribution to the literature of public finance, both because of its exposition of the difficulties in analyzing the present distribution of wholly tax exempt securities, and because of the effective way in which it states the arguments in behalf of and against tax exemption and demonstrates the utter futility of most of them.

RUFUS S. TUCKER

American Founders Trust, New York

Die Stabilisierung Der Mark, by Hjalmar Schacht. Berlin: Deutsche Verlags-Anstalt Stuttgart. 1927. 194 pp.

In this book Dr. Schacht presents the story of the reconstruction of German currency in a simple, straightforward and lucid manner.

Germany's war finance, which was based entirely on borrowing with a view to repayment by the conquered world, has been described before and not much new light on this period is thrown by Dr. Schacht's account. His description of post-war monetary developments is notable chiefly for the categorical statement that inflation was not a deliberate policy for the purpose of cheating foreign buyers of marks, but the inevitable result of demands for foreign payments made by the Reparation Commission. The heart of Schacht's story, however, centers around three dates: November 20, 1923, when the mark was stabilized; April 7, 1924, when the Reichsbank adopted its policy of credit restriction; and October 10, 1924, when the Reichsbank was reorganized with 800,000,000 marks raised by the international loan added to its reserves. It is during this period that Schacht demonstrated his outstanding characteristics of competence, confidence, and courage.

Prior to November, 1923, Schacht had been engaged in private business and during the immediately preceding years was head of the *Darmstädter und Nationalbank*. He had opposed the plan for establishing the rentenmark as a means for stabilizing German currency on the ground that mortgages back of a currency, no matter how adequate, do not assure convertibility and it is on convertibility that the stable value of a currency depends. Nevertheless, Schacht graciously admits that it was a stroke of genius on the part of Helfferich to create the rentenmark and thereby to arrest the course of inflation and to restore to the people a measure of confidence in their monetary unit. Notwithstanding his opposition to the rentenmark, Schacht was asked immediately after its creation to become Director of Currency, a post created *ad hoc*, with the privilege of participating in discussion at cabinet meetings, but without vote. Urged to make an immediate decision, Schacht accepted this tremendous responsibility, confident of his own ability to rehabilitate the currency and to persuade the Government to back up

his program. Schacht says that he accepted this responsibility with a serene mind, and in view of the desperateness of the situation one cannot but wonder what kind of a task it would have to be that would disturb his serenity!

Schacht's troubles began immediately, when he had to encounter a group of industrialists from the Rhine, headed by Hugo Stinnes, who wished to continue inflation, believing that to stop it at that time would be ruinous. Schacht stood firm and his hard-pressed compatriots, goaded by the French occupation and exasperated by his unwillingness to coöperate, informed the Minister of Finance that they could have nothing further to do with Mr. Schacht. The value of the mark at that time was approaching 3.9 trillions of marks to the dollar. Immediately following upon the conference with the Rhine industrialists, speculators threw enormous amounts of marks on the market and made a large volume of forward sales, with the consequence that between November 13 and November 26 the value of the dollar rose from 3.9 trillion marks to 11 trillion marks, but Schacht remained serene, and as the speculators were obliged to cover their commitments, they had to come to the Reichsbank for marks, with the consequence that the value of the dollar turned downward and by the 10th of December was 4.2 trillions, a figure at which it has remained ever since. This was Schacht's first victory and the first notable achievement in turning the tide of financial disorganization in Germany.

Soon after, Havenstein, the old Director of the Reichsbank, died, and the two leading candidates for the post were Helfferich and Schacht. The Board of Directors and the executive committee of the Reichsbank declared in favor of Helfferich and unanimously said that Schacht would not be acceptable. When asked whether the unanimous opposition by the Board of Directors would make any difference to him in undertaking the task, Schacht said that it would not. He was appointed, and pays tribute to the Board of Directors, who have given him loyal coöperation ever since. As soon as he became president, Schacht went to England and made informal arrangements for a loan of £5,000,000 and also made similar arrangements with a German syndicate. These £10,000,000 were to constitute the capital of a gold discount bank, which was to supply such foreign credits as would be necessary for the restoration of German industry and trade. It was at this time that Schacht first encountered the Dawes committee which to him appeared to be chiefly in the nature of an obstacle and a cause of delay in carrying out his program of financial reconstruction. He wanted to obtain the committee's approval of the immediate establishment of a gold bank, and when they hesitated, he made a public speech explaining that Germany could not wait for an indefinite number of months, while the Dawes plan was being constructed and brought into operation, and that immediate action was imperative. This appeal to public opinion was resented by the committee, but Schacht's proposal was accepted. It is characteristic of Schacht that in establishing the gold discount bank he insisted that it be given authority to issue notes, his secret purpose being to have a note issuing gold bank at his disposal in case the Dawes plan should not materialize. As it happened, Schacht fortunately never had occasion to use this power, but the coolness with which he planned for that eventuality commands one's admiration.

Schacht's principal objective before the committee of experts was to reduce the influence to be given to foreigners in the Reichsbank's management. He feels that he has accomplished his purpose, since the managing board consists entirely of Germans, and foreign participation is largely in an advisory capacity. Schacht's controversies with the experts were all on a high level and he thoroughly appreciates their endeavor to approach the problems arising from the war and from the peace in a realistic, economic spirit, rather than in a spirit of war-like nationalism. He speaks in a friendly manner of Sir Robert Kindersley and pays a high tribute to Owen D. Young.

As soon as his labors with the Dawes committee were completed, Schacht directed his undivided attention towards internal credit conditions. On April 9, 1924, he inaugurated a program of credit restriction, by issuing an order that after that date no additional credits would be granted by the Reichsbank, except to take the place of other credits that had expired. He fully recognizes that this is an arbitrary and unsatisfactory method of credit control and that only extreme necessity could justify such a procedure. He feels, however, that the necessity was critical and states that the distribution of credits was not as arbitrary as might at first glance appear, because credits were constantly maturing and it took but a short time to bring about an allotment of credit corresponding approximately to the importance of the borrowers to the Nation's economic welfare. There followed a terrible industrial crisis, mushroom growths of enterprises that had been fanned into life by inflation, fell in unbelievable numbers; Schacht was hated and denounced as a despot and a hangman, but normal credit conditions were gradually reëstablished in Germany, and now that the battle has been won Schacht feels that if he had to go through it again, he would adopt the same course of action.

The chapter on the reconstruction of the money and capital markets of Germany is more technical in nature than the others, and in it the author discusses many problems of credit policy. He buries the believers in managed currency under the weight of the word "*Indexwährungstheoretiker*," and yet expresses his own belief that the leading motive in credit policy should be social welfare.

Schacht's next crusade was against the loose granting and acceptance of foreign credits. He felt that Germany needed to go through a period of rigid economy and that the creation of foreign credits for non-essential, even though desirable, purposes would impede the country's recovery. It was not so much the volume of the credits that he scrutinized but the purposes for which they were used, and he succeeded in controlling these credits with a strong hand. Municipalities, as well as private enterprises, resented Schacht's interference, but he carried his point and thereby probably saved Germany from having a large volume of short-time foreign obligations that would have greatly handicapped her in her economic reconstruction.

The final chapter is entitled "International Coöperation," and in it Schacht, after declaring that nationalism is not a right but a duty, puts forward Germany's claim to have colonies, a claim that is very dear to his heart. He believes that sufficient markets for manufactured goods are in existence and refuses to listen to talk about shortage of markets "until the last savage in Africa has acquired a

radio apparatus." One can readily sympathize with Schacht's position that it is hard for a German to have all the other countries on earth considered fit to have colonies and to have Germany alone excluded from that field. There may, nevertheless, be some hesitancy in accepting his conviction that the establishment of colonies by Germany would materially help in increasing Germany's ability to pay reparations.

Schacht's book is a remarkable document, a candid tale of dramatic achievement, with no spirit of braggadocio, no bitterness or vindictiveness towards his opponents, and with a hopeful and progressive outlook for the future.

E. A. GOLDENWEISER

Washington, D. C.

Italy's International Economic Position, by Constantine E. McGuire. With the aid of the Council and Staff of the Institute of Economics. New York: The Macmillan Company. 1926. 588 pp.

This book is in many ways the best of the series of studies of post-war conditions in Europe brought out under the aegis of the Institute of Economics. There is, within its set limits, little of a fundamental character to which one can honestly object, and much to be praised. It is a straightforward, well-written account, concerned with describing the basic facts in the situation rather than with formulating any very elaborate explanations or solutions. Ostensibly an investigation simply of Italy's international economic position, the necessity of tracing the peculiarities in that position back to their domestic roots ends by making the book a study of the Italian national life and economic constitution as a whole. The result is in consequence a comprehensive and balanced survey of the country's aggregate economic organization and functioning, of a type that is all too rare. Such a synthesis is really the first step in the erection of a realistic body of economic analysis and doctrine, but few writers have hitherto had the courage or the patience to undertake it.

Italy, from the economic point of view, is a country placed in a curiously awkward and embarrassed position. On the one hand, she is extraordinarily poor in those natural resources which have become so vital to modern industrial civilization. She has a widely diversified agriculture, it is true, but despite an inordinate outlay of human effort can produce little more than two-thirds of the food she requires, and the proportion is more likely to fall in the future than to rise. She has little iron, still less coal, and practically no petroleum. Copper, tin, aluminum, and most of the other important minerals are found in insignificant quantities if at all. The sources of hydroelectric power are good, but are limited in extent and already extensively exploited. Little cotton is grown, even in the colonies. The most important non-agricultural resources of the country include such products as sulphur, nitrates and wood, but none of them are in the first rank of modern industrial requirements. On the other hand Italy has a relatively large and rapidly increasing population, now over forty millions in size and growing at the rate of nearly half a million a year. This population is progressive in its desires, and is seeking to take on all the forms of present-day industrial life and prosperity.

Italy's fundamental problem is therefore much the same in its general character as that which most of the other European countries face. She has a far larger population than she can feed from her own produce, and indeed the situation has been getting steadily worse ever since Italian industrialization began after the unification. The peculiar and crushing strain in Italy's position, however, arises from the fact that the solution which the other leading countries could adopt has not been open to her. Great Britain and France and Germany have exported their coal and iron and labor-power (and later their capital), either directly or by working up other products, and have received in return the foodstuffs and raw materials which they required. Italy, on the contrary, has little coal and iron, and almost no surplus capital. She is thus extraordinarily deficient in the main constituents of modern industrial life, although the pressure of population on her food supplies has compelled her to industrialize in order to live. But it has been a curious industrialization. She has had to import much of her coal, most of her iron and steel, and even the great majority of the less basic raw materials. Her chief and often her only major contribution to the manufacturing process has been simply her labor. Even so, if it had been possible for Italy to export all the manufactured goods made from imported materials an international surplus might have been achieved, of sufficient size to pay for the necessary food imports; but of course much of the manufactures were needed for domestic consumption. In the decade before the war the trade balance therefore showed a continuous deficit, averaging about two hundred million dollars a year—roughly a third of the total imports. It now runs nearly twice as high, in absolute terms, and is still of about the same relative size.

This trade deficit has been offset in various ways. In the decade before the war the two largest credit items were remittances received from Italians abroad, and expenditures of tourists. The two together covered about three-quarters of the deficit. But after 1905 these and all other "earned" items fell consistently short of the necessary total, and the result was both long-time borrowing and the appearance of a chronic floating indebtedness to foreigners on private account. By 1914 the floating debt totalled nearly a hundred million dollars, while the permanent foreign investment in Italian enterprises and government securities amounted to over half a billion dollars more—indebtedness which placed a steadily growing burden of interest charges on the Italian economy. Since the war the earned items of invisible income have inevitably fallen off, while on the other hand the excess of imports has increased enormously. The resulting international deficits have been almost paralyzing in size. In 1920, the worst year, the trade deficit alone amounted to over half a billion dollars, and the aggregate deficit of earned income to nearly four hundred millions. This was covered chiefly by new government borrowing abroad, and by adding to the private floating indebtedness. In more recent years, however, the position has improved materially, and in 1924 the international accounts for the first time showed an earned surplus.

But even apart from the presumably transitory disturbances incident to budget and currency instability, the underlying conditions governing Italian economic life are evidently of such character that they will make the international

position of the country extraordinarily difficult for many years to come. Before the war a chief source of invisible international income resulted from the emigration of individuals, who subsequently sent or brought back much of their earnings to their native homes. Since the war the United States, which was one of the principal destinations of the Italian emigration, has virtually closed its doors; Australia threatens similar measures; and Argentina and the European countries have a relatively limited capacity for further absorptions. Moreover, the post-war emigrants seem to move in family groups rather than as individuals, and to make their migration permanent, so that they no longer provide the characteristic pre-war stream of return remittances. Accompanying this probable future diminution in invisible international income is a probable increase in outgo on capital account, at least for a time, due chiefly to the foreign borrowing for industrial and municipal purposes which is now under way. In addition, any failure of reparations receipts to equal the interallied debt payments now scheduled will obviously increase the strain. The superficial answer to these fundamental problems is of course to increase commodity exportation relative to importation; but, as already indicated, that is precisely the thing that Italy's poverty of natural resources makes it extraordinarily difficult to do. Far more literally than in the frequently debated case of Germany, Italy can export only after first importing; and meanwhile the needs of her own population for foreign foodstuffs and raw materials are not only urgent, but are likely to grow even after all possible economies have been enforced. Mr. McGuire takes on the whole a sanguine view of the situation, and thinks that if no untoward conditions develop Italy will win through: that she will eventually acquire a permanent surplus of earned international income which can be applied to reducing her present foreign indebtedness. But it is clear that progress will be made only by a long series of small steps, and at the price of constant vigilance and effort.

This explanation of the origins and limitations of Italy's present economic position is the chief thesis of the book, and its most significant one. But the chapters on the country's currency and finance problems are also valuable and worth study, the more so since material on the subject is not easy to come by in this country. If one were to venture a criticism, it would simply be that the author has not gone far enough. He raises many interesting questions, and then, whether from modesty or purpose, turns without answering them to raise still others. There is hardly enough "what" in these chapters, and not nearly enough "why." So especially with respect to the relations of the Treasury to the banks, and of both to the foreign exchanges: the theorist is still left groping. It would have aided the reader greatly, too, if all the more or less indirect commentary on Fascism and the Fascist philosophy of economic life, which keeps cropping up as the *deus ex machina* in the history of the last few years, could have been summarized and brought together in one place. A rational account and defence of the Fascist régime, for which the book provides much of the material, would have been well worth undertaking. Finally, the distribution of space as between text and appendices is almost surely regrettable. The text is given less than half the book in number of pages, and hardly a third in number of words. The text is consequently simple and readable, but one frequently gets the

impression that the simplification and elimination have been carried to the point of producing mere thinness. It is too often necessary to plow through one or more long appendices to get at the real root of the matter.

These are criticisms of an essentially minor order, however, which do not impugn the general worth and merit of the book. As a whole, it is a comprehensive and well-done description of a singularly involved and baffling set of conditions; and it has value not only as a substantial contribution to recent economic history, but also as an example of the first step, too infrequently taken, in a realistic attack on the most intricate problems of economic life.

JAMES W. ANGELL

Columbia University

Rent Levels and Their Causes, by Joseph M. Gillman. Pittsburgh, Pennsylvania. 1926. xiii, 74 pp.

The determination of the levels of housing rents at any given time and place in relation to rent levels at some other time and place or in relation to the cost of other items of family expenditure should provide answers to many questions connected with the standard and cost of living. Any contribution, therefore, to the information available on the subject, either as regards method or fact is to be welcomed.

Rent levels may be found in a number of ways. How much families actually are paying in rent, or how much given lists of accommodations actually are rented for, give exact measurements; advertised prices, or listings with real estate brokers, supply bases for estimates. Each of these methods has been used by different students. In the present survey of rent levels and their causes in Pittsburgh, the investigator, who is chairman of the department of statistics and business research in the school of business administration of the University of Pittsburgh, resorted to the "For Rent" columns of the local newspapers as the best means of answering the question: "How much more rent did the average Pittsburgh family pay for average housing accommodations in 1923 than in 1913?" About 38,000 quotations for four, five and six-room unfurnished houses and apartments appearing in the Sunday edition of one of the Pittsburgh newspapers in the months of March, April, August, and September each year from 1913 to 1925, inclusive, were copied and analyzed. From study of these data, Dr. Gillman came to the conclusion that since 1913 average rents of housing in Pittsburgh had advanced 178.9 per cent, and median rents, 198.3 per cent. Except for a comparison of rents of housing in one well-known and clearly defined working class district with those in an area equally well accepted as the section of the city in which resided "the more prosperous business men, high-salaried executives, and professional men," no attempt was made, so far as can be told from the information presented in the report, to classify rents on the basis of number of rooms or type of housing.

Dr. Gillman believed that the rent index he had constructed was superior to that of the National Industrial Conference Board and of the United States Bureau of Labor Statistics because the two latter referred to housing of wage

earners' families only, and because, in the case of the Bureau's index, the method of following rents of individual property makes no allowance for deterioration and obsolescence, or for the influence of new building on rents, and because, in the case of the Board's index, chief reliance is placed on "estimates furnished by real estate offices." A comparison of his index with that of the Bureau of Labor Statistics led Dr. Gillman to the conclusion that "rents in Pittsburgh as disclosed in the present study have gone much higher than one would be led to believe from the government data" and "the use by the Bureau of Labor Statistics of the current index for rent in its general cost of living index involves a serious error." Unfortunately, however, in making this comparison, the Gillman rent index, with 1913 as 100, was plotted on an arithmetic scale from the same base line as the rent index of the Bureau, in which December, 1917, equals 100. The chart thus seems to show that while the former increased 178.9 per cent, the latter increased only 75.2 per cent. As a matter of fact, however, if the increase since 1917 shown by the newspaper data be compared with the rent increase shown by the Bureau of Labor Statistics figures over the same period, the newspaper data give an increase of 96 per cent in the general rent level, or 59 per cent in strictly working class neighborhoods, in which, presumably, the Bureau collected its rent quotations. Drawn on a ratio chart this error would not have been made but it is probably just as well that this was not done, inasmuch as, in the only ratio chart in the pamphlet, the distance from 100 to 200 is considerably greater than the distance from 200 to 400, which, of course, it should exactly equal.

The present writer is not convinced, however, that, even allowing for these errors, an analysis of newspaper quotations of rents affords a better measure of rent levels than do estimates made in other ways. If, for example, in the year to year comparisons more residences of four rooms or of five rooms or of six rooms were advertised than the year before or the year after, or if more brick, or frame or brick veneer houses were for rent in one year as compared with another; or if the demand for housing at some rents was so great that advertising was not so necessary in one year as in another, or that premiums were demanded; or if, on the other hand, conditions were such that discounts from advertised rates were allowed, the method of grouping all quotations advertised into a general average would result in figures which did not necessarily represent comparable rent conditions from year to year but, rather, figures which reflected the prevailing market for certain types of housing.

That this may well have been the case in this Pittsburgh rent survey is shown by some of the figures presented in the report in other connections. For example, in the spring of 1913, the accommodations advertised were as follows: four rooms, 33.2 per cent; five rooms, 32.4 per cent; six rooms, 34.4 per cent. In the spring of 1924, the advertisements were: four rooms, 39.5 per cent; five rooms, 36.0 per cent; six rooms, 24.5 per cent. The index was not weighted. Or, again, the number of permits for buildings issued changed so that while in 1920, 37.2 per cent were for brick construction, in 1925, only 21.3 per cent were for this type; in 1920, 37.7 per cent were for brick veneer while in 1925, 62.4 per cent were for that material. Frame buildings were relatively about as important in 1925 as in 1920. Furthermore, in 1918, there were only 1,021 houses and apartments

advertised; in 1919, 424; in 1920, 186; in 1921, 1,036, as contrasted with two or three thousand in other years, and 5,327 in 1925. That these and possibly other changes inevitably affected the number and kind of housing advertised, and therefore the rents quoted, is almost certain.

A large quantity of material was collected and analyzed regarding the level and causes of rents in Pittsburgh. The general impression given by this pamphlet is that an enormous amount of labor has been expended but that it serves little purpose either in establishing a more effective method of ascertaining rent changes than those already in use, or in establishing a more accurate measure of relative rent levels in Pittsburgh than has been computed for a number of years by the United States Bureau of Labor Statistics or the National Industrial Conference Board.

MARGARET LOOMIS STECKER

World War Debt Settlements, by Harold G. Moulton and Leo Pasvolsky, with the aid of the Council and Staff of the Institute of Economics, Washington, D. C. New York: The Macmillan Company. 1926. 435 pp.

It is the distinction, and, in our opinion, the outstanding merit of the books of the Institute, that in addition to presenting a comprehensive mass of data on the subject under discussion, they concentrate the reader's attention upon one dominant fact, and fix it indelibly in his mind. Out of the manifold problems of the reparation question, *Germany's Capacity to Pay* emphasized the difficulties of transferring payments from Germany to her creditors. Out of the innumerable problems of French finance, the Institute's study (*The French Debt Problem*) concentrated on the real burden of the internal debt.

Both of these books were controversial, in that, in addition to expounding a problem, they undertook to argue a thesis. The plan of *World War Debt Settlements* is different. There is no direct effort to prove that the American settlements are burdensome or light, much less is there any discussion as to the nature of the debts—whether they represent simple contract obligations, subsidies or contributions to a common cause. They are accepted as facts, and the object of the authors is to put these facts in their setting. To the average American, the title *World War Debt Settlements* suggests at first only the series of agreements between our treasury and its debtors. After reading this book, he realizes (unwillingly, perhaps) that the debts to America are only part of a snarl of indebtedness left by the war. There is no attempt to argue that our debtors' capacity to pay is conditioned by their receipts from Germany or their respective other debtors. The object of the book is merely to show the whole tangled picture.

This, then, is the salient impression of *World War Debt Settlements*. Legally and morally, we may deny relation between our debts and Germany's reparation payments; economically, they are an integral part of a whole. The same international financial machinery which handles the Allied payments to us, must handle Germany's payments to them and their payments to each other—and in addition carry the burden of transfers arising out of the ever growing volume of

international commercial debt. The book does not assert that the machinery cannot bear this load. It describes the machinery and appraises the load; the reader is entirely free to draw his own conclusions.

Its utility, to the average reader, lies in the quantity of specific data made available in condensed form. There is an excellent discussion of the debt position of the major countries (including the reparation debt) and a compact chapter on the "Extent of American Cancellations," well illustrated by graphic charts. Furthermore, two-thirds of the volume is devoted to appendices which reprint the essential documents relating to the reparation settlements, the texts of the American debt settlements (with schedules), of the British debt agreements with her debtors, and the Balfour note. Equipped with this volume, any one may feel himself armed to join the ever-growing group of controversialists, on whichever side he elects.

The text is terminated, so to speak, by an interrogation point. The final chapter, "Issues in Suspense," mentions briefly a number of disturbing questions —the relations of new borrowing to reparation and war debt payments, the "commercialization" of debt and reparation obligations, and the possibility of unfavorable effects from the debt payments upon our own production and trade.

R. B. WARREN

Security Speculation—The Dazzling Adventure, by Laurence H. Sloan. New York. Harper & Brothers. 1926. 280 pp.

The great changes in the economic position of our country and the reflection of these in the security market have directed the attention of economists and statisticians more than ever to the various phases of the stock market. Investigations have been made to determine and measure the relationships between interest rates, commodity prices, volume of production, distribution, and earnings to prices of securities.

Recent statistical studies have been made regarding the best methods to operate in the market. Common stocks have been compared with bonds as long term investments. Now, Laurence H. Sloan, in his book, *Security Speculation—The Dazzling Adventure*, goes one step farther and shows the advantages of long swing speculation in the stock market over the long pull method.

The long pull operation, according to the author, is one that is made with the idea of holding or adding to an investment in a particular stock for a term of years, usually ten to twenty years, regardless of the short or long swings in the stock market. The "long swing" refers particularly to the swing in stock prices of a year or more. In this operation stocks are purchased approximately at the bottom of the market and disposed of when the market reaches the next peak.

Five tests are applied to each of eighteen common stocks and fourteen dividend paying industrial preferred stocks. The first four of these tests show the results of the long pull operation.

Test 1 assumes that the speculator places a fixed sum of money in the stock market each year beginning with January 1, 1914, regardless of business, finan-

cial or market conditions, and sells out the entire holdings on January 1, 1926. Test 2 is also a long pull operation but instead of purchasing stock at the beginning of each year, the entire commitment is made at some specific time and held until January 1, 1926.

Three different periods are selected for these purchases: (a) about the 1914 low of the market; (b) at the mean price of the 1914–16 swing; and (c) at about the peak of the 1914–16 swing.

It is assumed that the funds are kept in the savings bank until purchases are made.

In the final test, which is for the long swing, it is assumed that the stock is purchased at or about the bottom of each major upswing in the market and disposed of at or about the peak of each movement, catching about two-thirds of the total swing. The proceeds of each operation are kept in the savings bank until the beginning of the next upswing.

The results of these tests show that over a period of years the purchase of securities for the long pull would have yielded an annual average profit as high as 32 per cent and as low as 7 per cent depending upon the time at which the purchases were made, and that with precisely the same group of eighteen industrial stocks but operating only in the long upswings of the market, the annual average profits would have been in excess of 73 per cent. In the case of the individual securities in the group, the results would have ranged from an average annual loss of 9 per cent to an annual average profit of 329 per cent. In the tests made for the preferred stock, the average annual gain ranged from 5.2 per cent to 8.4 per cent on the long pull operation, while on the long swing operation, the average gain was 12 per cent. For individual preferred stocks the annual average of profit ranged from 3.8 per cent to 15.7 per cent.

On a chart of stock prices for the period 1900 to the middle of 1926, the author marks off what he believes to be the beginning and end of each long swing of the market. For the purpose of his study, the period from January 1, 1914, to January 1, 1926, is selected. During this time there are four long upswings. The downward movements in prices are ignored in the study since it is assumed that the speculator is out of the market and has his funds in the savings bank. It is unfortunate that a longer period was not considered since the twelve years under study have been subject to so much wider fluctuations in prices than in the twelve years prior to 1914. The author admitted that it would have been more satisfactory to extend the period back farther. One of the obstacles cited was the lack of a major swing in prices from 1910 to 1913. The movements in these markets therefore were said to be inconclusive. While there may have been insufficient spectacular moves for the purposes of the tests, anyone examining the price movements of stocks in those years would have noted fluctuations of 15 to 50 points. United States Steel common, for example, one of the stocks used in the tests, ranged from a high of 91 in 1910 to a low price of 50 in 1913—a fluctuation of 41 points. Can this be considered an inconsequential move?

The author concludes from his tests that long swing operations offer possibilities of by far the larger financial gains, and that gratifying gains are possible in the long pull operation over a period of years providing that the securities are

bought at the end of a bear market and sold at a peak of a bull market. Another conclusion reached is that it is the exceptional security that returns an adequate profit if it is purchased at the wrong time, and correspondingly it is the exceptional security which fails to return an adequate profit if it is purchased at the right time. It was not necessary to make so many exhaustive or laborious calculations to arrive at such fundamentals.

The point is made that "contrary to the revered tradition, the average common stock does not yield its owner a higher return if it is held merely for income. It yields a higher return only when price appreciation is combined with dividend; in other words, the stock market itself when it is on a bull rampage will pay a much higher 'dividend' on the average speculative security than will the corporation behind the security." This conclusion, of course, is based on the eighteen stocks used in the tests, of which only seven paid dividends continuously during the entire period studied. Aside from this anyone who understands the difference between income and capital knows that the price of a stock will usually fluctuate in the market a greater amount than the annual dividend paid. For example, a stock paying $7 a share to yield 5 per cent would sell at 140. Increase the dividend to $8, and to obtain the same yield the stock would sell at 160. The dividend is increased $1, the principal, $20. Even with a constant dividend the price of a stock may be influenced by the changes in interest rates. Since by the rules laid down in his tests, the author did not give his hypothetical stockholder the benefit of subscription rights, the cumulative effect of dividends on the increased stocks is not known. This tends therefore to destroy the soundness of the calculations for the long pull.

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applied economic (substantive economics, money and banking, investments, political, industrial and legal history, social jurisprudence), which qualify the actuary to perform most of the work in economic, industrial and political statistics, which is being done today by persons without purposed training. Inspection of Mr. Menzler's paper will probably be helpful to the group of American teachers who plan during the coming summer to survey the aims and purposes of statistical teaching in American universities and colleges.

E. W. KOPF

Metropolitan Life Insurance Company

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